WORKING PAPER: COMMENTS WELCOME

Mortality Risk, Insurance, and the Value of Life*

Daniel Bauer
Georgia State University

Darius Lakdawalla
University of Southern California and NBER

Julian Reif
University of Illinois at Urbana-Champaign

Abstract. Public health insurance and public annuity programs account for half of federal spending, and have traditionally been viewed as unconnected. We develop an economic framework for valuing improvements in health and apply it to exploring the relationship between annuity programs and the value of life. Incorporating incomplete annuitization into the conventional economic theory of life-extension generates several novel findings. First, public annuity programs boost the demand for life-extension. For instance, retirement programs such as US Social Security add $150 billion to the aggregate value of a 1 percent decline in mortality. Second, in contrast to the conventional theory, a given mortality improvement is worth more to patients facing graver fatality risks. Holding income and wealth constant, we find that the value of statistical life (VSL) for a 50-year-old with 15 years of remaining life expectancy is $1 million greater than the VSL for a 50-year-old with 35 years of life expectancy. Finally, we introduce a new and more generalized concept, the value of statistical illness, which quantifies an individual's willingness to pay to avoid an increase in the risk of deterioration in her health state.

* We are grateful to Tatyana Deryugina, Nolan Miller, Alex Muermann, George Pennacchi, Mark Shepard, Dan Silverman, George Zanjani, and participants at the AEA/ARIA meeting and the NBER Insurance Program Meeting for helpful comments. Bauer acknowledges financial support from the Society of Actuaries. Lakdawalla discloses that he is the Chief Scientific Officer of Precision Health Economics (PHE) and an investor in its parent company, Precision Medicine Group. PHE provides research and consulting services to firms in the life sciences and health insurance industries.

I. INTRODUCTION

The economic analysis of risks to life and health has made enormous contributions to both academic discussions and public policy. Economists have used the standard tools of life-cycle consumption theory to propose a transparent framework that measures the value of improvements to both health and longevity (Arthur 1981; Rosen 1988; Murphy and Topel 2006). Economic concepts such as the value of statistical life now play central roles in public policy discussions surrounding investments in medical care, public safety, workplace safety, environmental hazards, and countless other arenas.

The standard framework has typically assumed complete annuitization and deterministic mortality risk. While analytically convenient and useful for illustrating some of the underlying economics, these assumptions hamper the model's predictive power in several ways when studying individual behavior and the relationships between alternative mechanisms for risk-reduction. In addition, they gloss over policy-relevant relationships between the value of life and the structure of the annuity market, and cannot meaningfully distinguish between preventive care and therapeutic care.

Complete annuity markets shield an individual against mortality risk. By the same logic, an incompletely annuitized consumer will have greater incentive to avoid or mitigate a sudden shock to mortality risk. A very simple example illustrates the intuition. Imagine a 60-year-old retiree with no bequest motive and a flat optimal consumption profile. If she fully annuitizes her savings, her consumption remains flat at, say, \$30,000 annually. Now suppose she cannot annuitize any of her wealth. It is well known that in this case it is optimal to shift consumption forward, because consumption allocated to later time periods will not be enjoyed in the event of an early death (see Figure 1). An important insight of our paper is that—at least for some initial period of time—mortality risk increases consumption, reduces the *marginal utility* of consumption, and thus increases the willingness to pay for life-extension. More generally, while mortality risks always reduce lifetime utility, the accompanying reduction in marginal utility can be large enough to cause the value of life to *increase*. This is in stark contrast to the conventional model with full annuitization, where an increase in mortality always reduces the value of life.

For the same underlying reason, the value of life varies with the size of mortality shocks when consumers are incompletely annuitized. This contrasts with the standard implication that a given reduction in mortality will be equally valuable, regardless of a consumer's baseline mortality risk. Instead, we show that the value of a statistical life is frequently higher for an individual diagnosed with a more fatal illness, and vice-versa. For example, we show using real-world data that, holding income and wealth constant, VSL varies from \$3.5 million to \$4.5 million for 50 year-olds with 35 to 15 years of remaining life expectancy, respectively. This insight is consistent with data on how consumers view the value of life-extension (Nord et al. 1995; Green and Gerard 2009; Linley and Hughes 2013). These findings can better inform the way health economists and healthcare payers assess the value of various medical technologies.

Our analysis illustrates the connections between public annuity programs and the societal value of mortality reductions. For example, we calculate that retirement programs such as Social Security have increased the aggregate value of reducing mortality risks by around 15%, so that a 1% reduction in population-wide mortality is \$150 billion more valuable than it would have been without retirement programs. And, perfectly completing the US annuity market would add a further \$100 billion of value to this mortality decline. Intuitively, annuitization tends to raise the value of life-extension at older ages where people might otherwise have outlived their wealth. Since the absolute number of deaths is quite high at those ages, this also tends to boost the value of proportional reductions in mortality.

Finally, our framework takes the more realistic perspective that an individual faces uncertainty over her future mortality risk. Allowing mortality to be stochastic produces additional insights. The conventional model quantifies the value of statistical lives, but it has little to say about the continuum of health events that precede death. Our framework lends itself naturally to a more general concept, the value of a statistical illness (VSI), which quantifies an individual's willingness to pay to avoid an increase in the risk of acquiring an illness that affects her mortality rate. This allows for the first time an economic comparison of the value of prevention to the value of treatment. In contrast to the conventional model, we show how the value of treatment technologies, which are used after an illness occurs, may differ from the value of preventive technologies, even when both increase life expectancy by identical amounts. This is because treatments are administered in worse health states than preventive investments.

Our study connects the vast literature on the value of life (Arthur 1981; Murphy and Topel 2006; Rosen 1988; Hall and Jones 2007) with the literature on annuities and life-cycle consumption models that goes back to Yaari (1965). It is well known that annuitization provides substantial value by insulating individuals from consumption risk. We show that it also increases the value of statistical life at older ages, and the value of mortality reductions in the aggregate.[1] Our results suggest that more attention should be paid to the public finance interactions between pension and healthcare systems.

Section II reviews the predictions of the conventional model for the returns to life-extension and demonstrates how relaxing the perfect annuity assumption alters the predictions of a model with deterministic mortality. Section III generalizes the framework further by incorporating the more realistic assumption of stochastic mortality. Section IV presents empirical analysis that: (1) quantifies how health shocks change the value of statistical life when annuity markets are incomplete; (2) illustrates how more severe health shocks cause consumers to place higher value on a given mortality reduction; (3) calculates the value of preventing different kinds of illness; and (4) estimates the effect of public annuity programs on the value of mortality risk-reduction. Section V concludes.

II. THE VALUE OF LIFE WHEN MORTALITY IS DETERMINISTIC

Consider an individual who faces a mortality risk. We are interested in analyzing the value of a marginal reduction in this risk. We first quantify this value in the conventional setting where markets are complete and the consumer has access to actuarially fair annuities (Rosen 1988; Murphy and Topel 2006). We then follow Shepard and Zeckhauser (1984) and repeat this exercise in a "Robinson Crusoe" economy where the consumer cannot purchase annuities to insure against her uncertain lifetime. We compare our findings for these two polar cases to illustrate the basic insights of the paper. Section III extends the model to accommodate stochastic mortality and medical interventions that stave off health decline. There we show that our conclusions are enriched but qualitatively unchanged in this more general setting.

Although it is optimal for a consumer to fully annuitize, real-world annuitization rates are quite low. This "annuity puzzle" is the subject of numerous papers. Many explanations have been suggested, but there is no consensus on what drives incomplete annuitization (Brown et al. 2008). Our model takes the low rate of annuitization as a given empirical fact and illustrates its significance for the value of life. Section IV uses a numerical model to probe the sensitivity of our results to different assumptions about consumer preferences, such as the presence of a bequest motive, which prior studies have argued might rationalize low observed rates of annuitization.

[1] Reichling and Smetters (2015) show that stochastic mortality and correlated medical costs can explain the puzzling observation that many households do not sufficiently annuitize their wealth. They take the positive correlation between health shocks and medical spending as a given. Our study sheds light on *why* these two phenomena are positively correlated.

We focus on improvements in longevity and their relationship to annuity insurance markets. We allow for improvements in quality of life as well, but we assume throughout the paper that individuals have access to complete indemnity healthcare insurance markets, as is standard in the literature. See Lakdawalla, Malani, and Reif (2017) for a detailed investigation of improvements in quality of life and their relationship to healthcare insurance markets.

II.A. The fully annuitized value of life

Let $c(t)$ be consumption at time t, W_0 be baseline wealth, $m(t)$ be exogenously determined income, ρ be the rate of time preference, and r be the rate of interest. Finally, define $q(t)$ as health-related quality of life at time t. Since it sacrifices little generality in our application, we take the life-cycle quality of life profile $q(t)$ as exogenous. As needed, one can consider any relevant quality of life profile in concert with a given profile of mortality. The maximum lifespan of a consumer is T, and her mortality (hazard) rate at any point in time is given by $\mu(t)$, where $0 \leq t \leq T$. The probability that a consumer will be alive at time t is:

$$S(t) = \exp\left[-\int_0^t \mu(s)ds\right]$$

At time $t = 0$, the consumer fully annuitizes. We assume that annuitization is actuarially fair.

The consumer's maximization problem is:

$$\max_{c(t)} \int_0^T e^{-\rho t} S(t) u(c(t), q(t)) dt$$
$$s.t.\, W(0) = W_0,$$
$$\int_0^T e^{-rt} S(t) c(t) dt = W(0) + \int_0^T e^{-rt} S(t) m(t) dt$$

The consumer's utility function, $u(c(t), q(t))$, depends on both consumption and health-related quality of life. We assume $u(\cdot)$ is strictly increasing, concave, and twice continuously differentiable. Let $u_c(\cdot)$ denote the marginal utility of consumption. Associating the multiplier θ with the wealth constraint, optimal consumption is characterized by the first-order condition:

$$e^{(r-\rho)t} u_c(c(t), q(t)) = \theta$$

To analyze the value of life, let $\delta(t)$ be a perturbation on the mortality rate with $\int_0^T \delta(t)dt = 1$, and consider

$$S^{\varepsilon}(t) = \exp\left[-\int_0^t (\mu(s) - \varepsilon\delta(s))ds\right], \text{where } \varepsilon > 0$$

Let $c^{\varepsilon}(t)$ represent the equilibrium variation in $c(t)$ caused by this perturbation. As shown in Rosen (1988), the marginal utility of this life-extension is given by

$$\begin{aligned}\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} &= \left.\frac{\partial}{\partial \varepsilon}\int_0^T e^{-\rho t} S^{\varepsilon}(t) u\big(c^{\varepsilon}(t), q(t)\big) dt\right|_{\varepsilon=0} \\ &= \int_0^T \left[e^{-\rho t} u(c(t), q(t)) + e^{-rt}\theta\big(m(t) - c(t)\big)\right]\left[\int_0^t \delta(s)ds\right] S(t) dt\end{aligned}$$

Dividing this result by the marginal utility of wealth, θ, then yields the marginal value of life-extension (VLE):

$$VLE = \int_0^T e^{-rt} S(t) \left(\frac{u(c(t), q(t))}{u_c(c(t), q(t))} + m(t) - c(t) \right) \left[\int_0^t \delta(s) ds \right] dt \tag{1}$$

It is also useful to characterize the value of a statistical life-year, which is the value of a one-period change in survival from the perspective of current time:

$$v(t) = \frac{u\big(c(t), q(t)\big)}{u_c\big(c(t), q(t)\big)} + m(t) - c(t) \tag{2}$$

The value of a statistical life-year is equal to the value of consumption in that year plus net savings, $m(t) - c(t)$. The net savings term is a consequence of the requirement that annuities be actuarially fair. The value of a life-year can be rewritten as:

$$v(t) = m(t) + c(t) \left(\frac{u\big(c(t), q(t)\big)}{c(t) u_c\big(c(t), q(t)\big)} - 1 \right) = m(t) + c(t)\phi(c, q)$$

where $\phi(c, q)$ represents the consumer surplus value per unit of consumption. It is positive if average utility exceeds marginal utility. A life-year adds value through two different channels: an increase in earnings, which can finance additional consumption, and an increase in consumer surplus.[2]

A canonical choice for $\delta(\cdot)$ in (1) is the Delta-Dirac function, so that the mortality rate is perturbed at $t = 0$ and remains unaffected otherwise. This then yields an expression that is commonly called the value of a statistical life (VSL):

$$VSL \equiv \int_0^T e^{-rt} S(t) v(t) dt \tag{3}$$

VSL corresponds to the value that the individual places on a marginal reduction in risk of death in the current period. For example, it is the amount that 1,000 people would be collectively willing to pay to eliminate a current risk that is expected to kill one of them. It is equal to the present discounted value of lifetime consumption, plus the change in net savings.

The value of statistical life depends on consumption and the quality of life. Define the elasticity of intertemporal substitution as:

$$\frac{1}{\sigma} \equiv -\frac{u_{cc} c}{u_c}$$

In addition, define the elasticity of quality of life with respect to the marginal utility of consumption as:

$$\eta \equiv \frac{u_{cq} q}{u_c}$$

When this term is positive, the marginal utility of consumption is higher in healthier states, and vice-versa.

[2] Positive consumer surplus may require that consumption remain above a "subsistence" level, $\underline{c} > 0$.

Taking logarithms of the first-order condition for consumption and differentiating with respect to time yields the rate of change for consumption over the life cycle:

$$\frac{\dot{c}}{c} = \sigma(r-\rho) + \sigma\eta\frac{\dot{q}}{q} \tag{4}$$

If one assumes that $r > \rho$, and that the marginal utility of consumption is higher when health status is better, then life-cycle consumption will have the inverted U-shape observed in real-world data.[3]

Note the crucial feature of the conventional model that consumption growth over the life-cycle is independent of mortality risk, because the individual is fully insured against that risk. This feature in turn implies that the rate of change in the value of a life-year is also not a function of mortality risk:

$$\frac{\dot{v}}{v} = \left(\frac{1}{\sigma v}\frac{u}{u_c}\right)\frac{\dot{c}}{c} + \left(\frac{-\eta}{v}\frac{u}{u_c} + \frac{q}{v}\frac{u_q}{u_c}\right)\frac{\dot{q}}{q} + \frac{\dot{m}}{v}$$

Although mortality risk has no effect on the rate of change in the value of a statistical life-year when consumers are fully annuitized, inspection of equation (3) reveals that an increase in mortality affects the value of life through two related channels. First, it reduces the average value of a life-year, $v(t)$, because higher mortality forces the individual to consume at a higher rate, thereby lowering the consumer surplus value per unit of consumption, $\phi(c,q)$. Second, it reduces the probability, $S(t)$, that the individual will survive to enjoy consumption and earn money in a particular year. This leads to the conventional wisdom that, all else equal, VSL is lower for individuals with higher mortality risk.

II.B. The uninsured value of life

To illustrate the effects of annuitization, we consider a model without any annuitization possibilities. In our calibration exercises later, we will consider various partial annuitization schemes. To characterize the model without annuitization, we employ the Yaari (1965) model of consumption behavior under mortality risk. The consumer's maximization problem is:

$$\max_{c(t)} \int_0^T e^{-\rho t} S(t) u(c(t), q(t)) dt$$

$$s.t. W(0) = W_0,$$

$$W(t) \geq 0, W(T) = 0,$$

$$\dot{W} = rW(t) + m(t) - c(t)$$

If the non-negative wealth constraint binds, then the solution to the consumer's problem is to set $c(t) = m(t)$. Otherwise, the solution is to maximize subject to the constraint on the law of motion for wealth. We focus here on the latter, nontrivial case.

The consumer's first-order condition for consumption is:

$$e^{(r-\rho)t} S(t) u_c(c(t), q(t)) = \theta$$

[3] Consumption climbs early in life as the benefits to savings diminish. It declines later in life when quality of life deteriorates. This inverted U-shape for the age profile of consumption has been widely documented across different countries and goods (Carroll and Summers 1991; Banks et al. 1998; Fernandez-Villaverde and Krueger 2007).

Unlike in the case of perfect markets, the survival function enters the consumer's first-order condition for optimal consumption. Instead of setting the discounted marginal utility of consumption equal to the marginal utility of wealth, the consumer sets the *expected* discounted marginal utility of consumption at time t equal to the marginal utility of wealth. This effectively shifts consumption to earlier ages in the life-cycle. This is rational because consumption allocated to later time periods will not be enjoyed in the event of an early death.

The expression for the marginal utility of life extension is:

$$\begin{aligned}
\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} &= \left.\frac{\partial}{\partial \varepsilon}\int_0^T e^{-\rho t} S^{\varepsilon}(t) u\big(c^{\varepsilon}(t), q(t)\big)dt\right|_{\varepsilon=0} \\
&= \int_0^T e^{-\rho t}\left[\int_0^t \delta(s)ds\right] S(t)u(c(t), q(t))dt + \int_0^T e^{-\rho t} S(t) u_c(c(t), q(t)) \left.\frac{\partial c^{\varepsilon}(t)}{\partial \varepsilon}\right|_{\varepsilon=0} dt \\
&= \int_0^T e^{-\rho t}\left[\int_0^t \delta(s)ds\right] S(t)u(c(t), q(t))dt + \theta \frac{\partial}{\partial \varepsilon}\int_0^T e^{-rt} c^{\varepsilon}(t)dt \\
&= \int_0^T e^{-\rho t}\left[\int_0^t \delta(s)ds\right] S(t)u(c(t), q(t))dt,
\end{aligned}$$

where the last equality follows from application of the budget constraint.[4]

Dividing this result by the marginal utility of wealth, θ, then yields the marginal value of life-extension (VLE):

$$VLE = \int_0^T e^{-\rho t}\left[\int_0^t \delta(s)ds\right] S(t)\frac{u\big(c(t), q(t)\big)}{u_c\big(c(0), q(0)\big)}dt = \int_0^T e^{-rt}\left[\int_0^t \delta(s)ds\right]\frac{u\big(c(t), q(t)\big)}{u_c\big(c(t), q(t)\big)}dt \tag{5}$$

In this setting, the value of a life-year from the perspective of current time is:

$$v(t) = \frac{u\big(c(t), q(t)\big)}{u_c\big(c(t), q(t)\big)} \tag{6}$$

When the consumer is uninsured, the value of a life-year depends only on the value of consumption. The net savings term is absent in equation (6) because life-extension has no effect on the consumer's budget constraint.

Choosing again the Delta-Dirac function for $\delta(\cdot)$ yields an expression for VSL that differs from the perfect markets case:

$$VSL = \int_0^T e^{-rt} v(t)dt \tag{7}$$

The value of statistical life is proportional to the expected discounted (lifetime) utility of consumption, and inversely proportional to the marginal utility of consumption. It is well known that removing annuity markets lowers lifetime utility (Yaari 1965). As we shall show more formally below, removing these markets also shifts consumption to earlier ages, thereby lowering the marginal utility of consumption, at

[4] The budget constraint $W(T) = 0$ implies $\int_0^T e^{-rt} c^{\varepsilon}(t)dt = W_0 + \int_0^T e^{-rt} m(t)dt$, which does not depend on survival and thus is unaffected by life extension.

least at those ages. Thus, the effect of annuity markets on VSL is in general ambiguous. In other words, exposure to mortality risk does not necessarily lower VSL. When consumers shift consumption forward, the near-term life-years rise in value but distant life-years go down in value. In the next section, we will show that this basic insight extends to exposing a consumer to a mortality "shock". We emphasize that in both cases the result depends critically on whether consumers are fully annuitized.

Unlike the perfect markets case, the life-cycle consumption profile of the non-annuitized individual depends explicitly on mortality risk. Taking logarithms of the first-order condition for consumption and differentiating with respect to time yields:

$$\frac{\dot{c}}{c} = \sigma(r-\rho) + \sigma\eta\frac{\dot{q}}{q} - \sigma\mu(t) \tag{8}$$

Comparing this result to the standard case, given by equation (4), reveals both similarities and differences. As in the standard, fully annuitized model, the non-annuitized consumption profile described by equation (8) changes shape when the rate of time preference is above or below the rate of interest and when the quality of life changes. Unlike in the standard model, however, the consumption profile described by equation (8) depends explicitly on the mortality rate, $\mu(t)$. Higher rates of mortality depress the rate of consumption growth over the life-cycle. This rate of growth is always higher in the fully annuitized case, in which the last term drops out of the consumption growth equation (8). Put another way, removing the annuity market "pulls consumption earlier" in the life-cycle.

An appealing feature of the uninsured model is that it generates an inverted U-shape for the profile of consumption under quite natural assumptions. In particular, low income early in life and high mortality risk later in life are sufficient conditions for the inverted U-shape consumption profile. One need not impose the ad hoc assumptions on the signs of $r-\rho$ or η that are necessary in the fully annuitized model (Murphy and Topel 2006).

The life-cycle profile of the value of a statistical life-year is:

$$\frac{\dot{v}}{v} = \left(\frac{1}{\sigma} + \frac{c}{v}\right)\frac{\dot{c}}{c} + \left(\frac{qu_q}{u} - \eta\right)\frac{\dot{q}}{q} \tag{9}$$

An important implication of (9) is that willingness to pay for longevity depends on the life-cycle mortality profile because of its dependence on the rate of change in consumption. Holding quality of life constant, it is evident from equation (6) that increases in the mortality rate will raise v, the value of a statistical life-year. That is, mortality will shift forward the value of life. At the individual level, consumers facing highly fatal diseases – e.g., cancer – will pay more for a marginal life-year than healthy peers, all else equal. This differs from the implications of the conventional model, in which the onset of illness has no impact in itself on the value of a statistical life-year. At the aggregate level, as societies become richer and live longer, the fraction of wealth spent on health will depend not just on the income elasticity of health, but also on the degree of survival uncertainty they face. We return to this point in our empirical exercise. Furthermore, our results imply that public programs such as Social Security that increase annuitization levels will affect society's willingness to pay for longevity, thereby creating a feedback loop that could dampen or increase program expenditures.[5] As a general matter, the model demonstrates that the degree of annuitization influences how people value gains in longevity.

[5] Philipson and Becker (1998) make the important, but distinct, point that the moral hazard effects of public annuity programs also increase an individual's willingness to pay for longevity gains.

In the next section, we allow mortality to be stochastic so that we can investigate the value of prevention and the effect of health shocks on the value of life. Before turning to that analysis, we pause to note that suffering a health shock is similar to removing access to annuity markets, which exposes an individual to mortality risk. This shifts the value of a life-year forward. The net effect on VSL is ambiguous. As we shall see, health shocks have a similar effect.

III. THE VALUE OF LIFE WHEN MORTALITY IS STOCHASTIC

The previous analysis demonstrates that mortality risk affects the value of life when annuity markets are incomplete. Prior studies have overlooked this relationship by assuming complete annuitization. However, the conventional framework is ill-equipped to study the influence of mortality risk for another reason as well. Prior analysis, just like our model above, treats the mortality rate as a nonrandom parameter (cf. Murphy and Topel, 2006). Thus, shifts in mortality risk reflect preordained and anticipated changes in mortality. In the real world, however, neither the timing nor the size of shifts in mortality risk is known. As a related matter, the conventional framework does not allow for different health states. This omission precludes a meaningful analysis of the value of preventing health deterioration.

This section extends our analysis to allow for stochastic mortality. Specifically, we assume that the mortality rate now depends on the individual's health state. Let Y_t be a continuous-time Markov chain with finite state space $Y = \{1,2, \dots n\}$.[6] Denote the transition intensities by:

$$\lambda_{ij}(t) = \lim_{h \to 0} \frac{1}{h} \mathbb{P}[Y_{t+h} = j | Y_t = i], j \neq i,$$

$$\lambda_{ii}(t) = -\sum_{j \neq i} \lambda_{ij}(t)$$

The mortality rate at time t is defined as

$$\mu(t) = \sum_{j=1}^{n} \overline{\mu}_j(t) \mathbf{1}\{Y_t = j\}$$

where $\left\{\overline{\mu}_j(t)\right\}$ are exogenous and $\mathbf{1}\{Y_t = j\}$ is an indicator variable equal to 1 if the individual is in state j at time t and 0 otherwise. Without much loss of generality, we assume that individuals can transition only to higher-numbered states, i.e., $\lambda_{ij}(t) = 0\ \forall j < i$, so that the probability that a consumer in state i at time 0 remains in state i at time t is equal to:[7]

$$\tilde{S}(i,t) = \exp\left[-\int_0^t \overline{\mu}_i(s) + \sum_{j \neq i} \lambda_{ij}(s)\, ds\right]$$

III.A. The fully annuitized value of life

Even when mortality is stochastic, a complete annuities market allows the consumer to fully insure against mortality risk. We assume a full menu of actuarially fair annuities is available where consumers

[6] The finite state assumption is only a mild restriction because of the approximation property of Markov chains.

[7] That is, an individual can transition from state $i < j$ but not vice versa. This does not meaningfully limit the generality of our model, because one can always define a new state $k > j$ where $\overline{\mu}_k(t) = \overline{\mu}_i(t) \forall t$.

can choose consumption streams, $c_{Y_t}(t)$, that depend on the health state, Y_t. The consumer's maximization problem is:

$$\max_{c_{Y_t}(t)} \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) u\big(c_{Y_t}(t), q_{Y_t}(t)\big) dt \,\middle|\, Y_0\right] \tag{10}$$

$$\text{s.t.}\, \mathbb{E}\left[\int_0^T e^{-rt} S(t) c_{Y_t}(t) dt \,\middle|\, Y_0\right] = \mathbb{E}\left[W_0 + \int_0^T e^{-rt} S(t) m_{Y_t}(t) dt \,\middle|\, Y_0\right] \equiv \overline{W}(0, Y_0)$$

where the net present value of wealth and future earnings at time t in state i is $\overline{W}(t,i)$, and $S(t)$ is defined as before. Define the consumer's objective function at time u as:

$$J(u,i) = \mathbb{E}\left[\int_0^{T-u} e^{-\rho t} \exp\left\{-\int_0^t \mu(u+s) ds\right\} u\big(c_{Y_{u+t}}(u+t), q_{Y_{u+t}}(u+t)\big) dt \,\middle|\, Y_u = i\right] \tag{11}$$

We can write the objective function (11) recursively as:

$$J(u,i) = \int_0^{T-u} e^{-\rho t} \exp\left\{-\int_0^t \bar{\mu}_i(u+s) + \sum_{j \neq i} \lambda_{ij}(u+s)\, ds\right\} \left(u(c_i(u+t), q_i(u+t)) + \sum_{j \neq i} \lambda_{ij}(u+t) J(u+t, j) \right) dt$$

Similarly, current wealth at time u in state i, including the value of future labor income, pays for future consumption such that:

$$\begin{aligned} \overline{W}(u,i) &= \mathbb{E}\left[\int_0^{T-u} e^{-rt} \exp\left\{-\int_0^t \mu(u+s) ds\right\} c_{Y_{u+t}}(u+t) dt \,\middle|\, Y_u = i\right] \\ &= \int_0^{T-u} e^{-rt} \exp\left\{-\int_0^t \bar{\mu}_i(u+s) + \sum_{j \neq i} \lambda_{ij}(u+s)\, ds\right\} \left(c_i(u+t) + \sum_{j \neq i} \lambda_{ij}(u+t)\, \overline{W}(u+t, j) \right) dt \end{aligned}$$

This in turn implies

$$\frac{\partial \overline{W}(t,i)}{\partial t} = \big(r + \bar{\mu}_i(t)\big) \overline{W}(t,i) - c_i(t) + \sum_{j \neq i} \lambda_{ij}(t) \big[\overline{W}(t,i) - \overline{W}(t,j)\big]$$

Define the optimal value function as

$$V\big(t, \overline{W}_t, Y_t\big) = \max_{\{c_{Y_s}(s), s \geq t\}} \{J(t, Y_t)\}$$

where $\overline{W}_t = \left(\overline{W}(t, Y_1), \ldots \overline{W}(t, Y_n)\right)$. Under conventional regularity conditions, we know that if V and its partial derivatives are continuous, then V satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$
\begin{aligned}
&\left(\rho+\bar{\mu}_i(t)\right)V\left(t,\overline{W}_t,i\right) \qquad (12)\\
&\quad=\max_{c_i(t)}\left\{u\left(c_i(t),q_i(t)\right)\right.\\
&\quad+\sum_{k=1}^{n}\frac{\partial V\left(t,\overline{W}_t,i\right)}{\partial\overline{W}(t,k)}\left[\left(r+\bar{\mu}_k(t)\right)\overline{W}(t,k)-c_k(t)+\sum_{l\neq k}\lambda_{kl}(t)\left[\overline{W}(t,k)-\overline{W}(t,l)\right]\right]\\
&\quad\left.+\frac{\partial V(t,\overline{W}_t,i)}{\partial t}+\sum_{j\neq i}\lambda_{ij}(t)\left[V(t,\overline{W}_t,j)-V\left(t,\overline{W}_t,i\right)\right]\right\},1\leq i\leq n
\end{aligned}
$$

We are interested in understanding how optimal consumption, and thus the value of life, changes over the life-cycle in this problem. We follow Parpas and Webster (2013), who demonstrate that it is possible to reformulate a stochastic optimization problem as a deterministic problem that takes $V\left(t,\overline{W}_t,j\right),j\neq i$, along with the corresponding optimal policies, as exogenous. This then allows us to apply the maximum principle and derive analytic expressions.

Lemma 1:

The optimal value function for $Y_0=i$, $V\left(t,\overline{W}_t,i\right)$, for the following deterministic optimization problem also satisfies the HJB given by (12), for each $i\in\{1,\dots,n\}$:

$$
V_0\left(0,\overline{W}_0,i\right)=\max_{c_i(t)}\left[\int_0^T e^{-\rho t}\tilde{S}(i,t)\left(u(c_i(t),q_i(t))+\sum_{j\neq i}\lambda_{ij}(t)V\left(t,\overline{W}_t,j\right)\right)dt\right] \qquad (13)
$$

$$
\text{s.t.}\frac{\partial\overline{W}(t,j)}{\partial t}=\left(r+\bar{\mu}_j(t)\right)\overline{W}(t,j)-c_j(t)+\sum_{k\neq j}\lambda_{jk}(t)\left[\overline{W}(t,j)-\overline{W}(t,k)\right],j=1,\dots,n
$$

where $V\left(t,\overline{W}_t,j\right)$ and $c_j(t),j\neq i$, are taken as exogenous.

Proof of Lemma 1: see appendix

Following Bertsekas (2005), the Hamiltonian for the (deterministic) maximization problem (13) is:

$$
\begin{aligned}
H\left(\overline{W}_t,c_i(t),p_t\right)&=e^{-\rho t}\tilde{S}(i,t)\left(u(c_i(t),q_i(t))+\sum_{j\neq i}\lambda_{ij}(t)V\left(t,\overline{W}_t,j\right)\right) \qquad (14)\\
&+\sum_{k=1}^{n}p_t^{(k)}\left[(r+\bar{\mu}_k(t))\overline{W}(t,k)-c_k(t)+\sum_{l\neq k}\lambda_{kl}(t)\left[\overline{W}(t,k)-\overline{W}(t,l)\right]\right]
\end{aligned}
$$

where $p_t^{(k)}$ is the adjoint variable corresponding to wealth $\overline{W}(t,k)$.

Lemma 2:

The consumer's first-order condition for the Hamiltonian (14) for $Y_o=i$ is

$$
e^{(r-\rho)t}u_c(c_i(t),q_i(t))=\theta \qquad (15)
$$

where $\theta=\partial V\left(0,\overline{W}_0,i\right)/\partial\overline{W}(0,i)$ is equal to the marginal utility of wealth.

Proof of Lemma 2: see appendix

To analyze the value of life, we again let $\delta(t)$ be a perturbation on the mortality rate with $\int_0^T \delta(t)dt = 1$. As in the deterministic case, we will first derive the marginal utility of the life extension associated with this perturbation.

Proposition 3:

The marginal utility of life extension takes the same form as in the deterministic case:

$$\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} = \mathbb{E}\left[\left.\int_0^T \left[e^{-\rho t}u\big(c_{Y_t}(t), q_{Y_t}(t)\big) + e^{-rt}\theta(m_{Y_t}(t) - c_{Y_t}(t))\right]\left(\int_0^t \delta(s)ds\right)S(t)dt\right| Y_0\right]$$

Proof of Proposition 3:

The marginal utility of life extension is defined as

$$\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} = \frac{\partial}{\partial \varepsilon}\mathbb{E}\left[\left.\int_0^T e^{-\rho t}\exp\left\{-\int_0^t \mu(s) - \varepsilon\delta(s)ds\right\}\left(u\left(c^{\varepsilon}_{Y_t}(t), q_{Y_t}(t)\right)\right)dt\right| Y_0\right]\Bigg|_{\varepsilon=0}$$

where $c^{\varepsilon}(t)$ represents the equilibrium variation in $c(t)$ caused by this perturbation. Then

$$\begin{aligned}\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} &= \mathbb{E}\left[\left.\int_0^T e^{-\rho t}\left(\int_0^t \delta(s)ds\right)\exp\left\{-\int_0^t \mu(s)ds\right\}u\big(c_{Y_t}(t), q_{Y_t}(t)\big)dt\right| Y_0\right] \\ &\quad + \mathbb{E}\left[\left.\int_0^T e^{-\rho t}\exp\left\{-\int_0^t \mu(s)ds\right\}u_c\left(c^{\varepsilon}_{Y_t}(t), q_{Y_t}(t)\right)\left.\frac{\partial c^{\varepsilon}_{Y_t}(t)}{\partial \varepsilon}\right|_{\varepsilon=0}dt\right| Y_0\right] \\ &= \mathbb{E}\left[\left.\int_0^T e^{-\rho t}\left(\int_0^t \delta(s)ds\right)\exp\left\{-\int_0^t \mu(s)ds\right\}u\big(c_{Y_t}(t), q_{Y_t}(t)\big)dt\right| Y_0\right] \\ &\quad + \theta\mathbb{E}\left[\left.\int_0^T e^{-rt}\exp\left\{-\int_0^t \mu(s)ds\right\}\left.\frac{\partial c^{\varepsilon}_{Y_t}(t)}{\partial \varepsilon}\right|_{\varepsilon=0}dt\right| Y_0\right]\end{aligned}$$

Finally, the budget constraint implies

$$\begin{aligned}0 = \left.\frac{\partial W_0}{\partial \varepsilon}\right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon}\mathbb{E}\left[\left.\int_0^T e^{-rt}\exp\left\{-\int_0^t \mu(s) - \varepsilon\delta(s)ds\right\}\left(c^{\varepsilon}_{Y_t}(t) - m_{Y_t}(t)\right)dt\right| Y_0\right]\Bigg|_{\varepsilon=0} \\ &= \mathbb{E}\left[\left.\int_0^T e^{-rt}\left(\int_0^t \delta(s)ds\right)\exp\left\{-\int_0^t \mu(s)ds\right\}\left(c_{Y_t}(t) - m_{Y_t}(t)\right)dt\right| Y_0\right] \\ &\quad + \mathbb{E}\left[\left.\int_0^T e^{-rt}\exp\left\{-\int_0^t \mu(s)ds\right\}\left.\frac{\partial c^{\varepsilon}_{Y_t}(t)}{\partial \varepsilon}\right|_{\varepsilon=0}dt\right| Y_0\right]\end{aligned}$$

Plugging this last result into the expression for $\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0}$ then yields the desired result.

QED

Choosing again the Delta-Dirac function for $\delta(\cdot)$ and dividing the result by the marginal utility of wealth, θ, yields the value of statistical life:

$$VSL = \mathbb{E}\left[\left.\int_0^T e^{-rt}S(t)v_{Y_t}(t)dt\right| Y_0\right] \tag{16}$$

where the value of a statistical life-year is:

$$v_{Y_t}(t) = \frac{u\left(c_{Y_t}(t), q_{Y_t}(t)\right)}{u_c\left(c_{Y_t}(t), q_{Y_t}(t)\right)} + m_{Y_t}(t) - c_{Y_t}(t)$$

Comparing (16) to (3) reveals that stochastic mortality does not alter the basic expression for VSL. Consumers continue to discount future life-years by the rate of interest and by survival. One notable difference is that stochastic mortality generates variance in the value of life, which can now increase or decrease following the transition to a new health state.

We can obtain the life-cycle profile of consumption by differentiating the first-order condition (15) with respect to t. Doing so confirms that, as in the deterministic case, annuitization insulates consumption from mortality risk:[8]

$$\frac{\dot{c}_{Y_t}}{c_{Y_t}} = \frac{\dot{c}}{c} = \sigma(r-\rho) + \sigma\eta\frac{\dot{q}}{q}$$

Our results demonstrate that stochastic mortality, by itself, does not alter the basic insights regarding VSL offered by the prior literature as long as one maintains the assumption of full annuitization.

A novel feature of the stochastic model is that it permits an investigation into the value of prevention, i.e., the value of a reduction in the probability of transitioning to a different health state. This is not possible in a deterministic environment, where there is implicitly only one health state.

To analyze the value of prevention, let $\delta_{ij}(t)$ be a perturbation on $\lambda_{ij}(t)$, where $\sum_{j\neq i}\int_0^T \delta_{ij}(t)dt = 1$. As in the life-extension case, it is helpful to choose the Delta-Dirac function for $\delta(\cdot)$, so that the probability is perturbed at $t = 0$ and remains unaffected otherwise. It is also helpful to consider a reduction in the transition probability for only one alternative state, j_0, so that $\delta_{ij}(t) = 0\ \forall j \neq j_0$.

Proposition 4:

Define the value of statistical illness, $VSI(i, j_0)$, to be the value of marginal reduction in the probability of transitioning to state j_0 when in state i. This value is equal to:

$$\begin{aligned} VSI(i, j_0) = {} & \mathbb{E}\left[\int_0^T e^{-rt}\left[\frac{u\left(c_{Y_t}(t), q_{Y_t}(t)\right)}{u_c\left(c_{Y_t}(t), q_{Y_t}(t)\right)} + m(t) - c(t)\right]S(t)dt\,\middle|\, Y_0 = i\right] \\ & - \mathbb{E}\left[\int_0^T e^{-rt}\left[\frac{u\left(c_{Y_t}(t), q_{Y_t}(t)\right)}{u_c\left(c_{Y_t}(t), q_{Y_t}(t)\right)} + m(t) - c(t)\right]S(t)dt\,\middle|\, Y_0 = j_0\right] \\ & = VSL(i) - VSL\left(j_0 \middle| \overline{W}(0) = W^*\right) \end{aligned} \tag{17}$$

where W^* is the value of the annuity that was initially purchased in state i that promised the state-contingent consumption stream $c^*_{Y_t}(t)$:

[8] As discussed earlier, we assume—like all prior studies—that full indemnity healthcare insurance is available, which is equivalent to assuming that $q(t)$ is independent of the health state. Without this assumption, sudden decreases in q could cause the value of life to jump (Lakdawalla, Malani, and Reif 2017).

$$W^{*}=\mathbb{E}\left[\int_{0}^{T}e^{-rt}S(t)c_{Y_t}^{*}(t)dt\,\middle|\,Y_0=j_0\right]$$

Proof of Proposition 4: see appendix

The notation in equation (17) indicates that VSL in state j_0 is evaluated under the assumption that the consumer's annuity was purchased when she was in state i. If life expectancy in state j_0 is lower than in state i, the value of the annuity to the consumer falls.

Notice that if state j_0 is death, then VSI simplifies to VSL (see equation (16)). It is straightforward to generalize equation (17) to include a treatment that prevents multiple diseases: in that case, VSI is equal to VSL in the healthy state minus the weighted average of the VSL's across the multiple disease states.

This result is consistent with the notion that, all things equal, it is more valuable to prevent serious diseases than mild diseases. More generally, (17) provides justification for the common practice of equating the values of prevention and treatment. For example, conventional cost-effectiveness frameworks value a treatment that prevents the onset of an illness that lowers life expectancy by 10 years the same as a therapeutic treatment that cures an illness and adds 10 years of life expectancy (Drummond et al. 2005). As we shall see, this equivalence breaks down when the consumer is no longer fully annuitized.

III.B. The uninsured value of life

The consumer's maximization problem is:

$$\max_{c_{Y_t}(t)}\mathbb{E}\left[\int_{0}^{T}e^{-\rho t}S(t)u\big(c_{Y_t}(t),q_{Y_t}(t)\big)dt\,\middle|\,Y_0\right]$$

$$\begin{aligned}\text{s.t.}\,&W(0)=W_0,\\ &W(t)\geq 0, W(T)=0,\\ &\dot{W}=rW(t)+m_{Y_t}(t)-c_{Y_t}(t)\end{aligned}$$

As in the deterministic model presented in Section II.B, we will focus on the non-trivial case where the non-negative wealth constraint does not bind. We proceed as in the fully annuitized stochastic mortality case and analogously define an optimal value function $V(t,W(t),i)$, which satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\begin{aligned}&\big(\rho+\overline{\mu}_i(t)\big)V(t,W(t),i)\\ &\quad=\max_{\{c_i(t)\}}\left\{u\big(c_i(t),q_i(t)\big)+\frac{\partial V(t,W(t),i)}{\partial W(t)}[rW(t)+m(t)-c_i(t)]\right.\\ &\quad\left.+\frac{\partial V(t,W(t),i)}{\partial t}+\sum_{j\neq i}\lambda_{ij}(t)[V(t,j)-V(t,i)]\right\},i=1,\dots,n\end{aligned}\tag{18}$$

As in the fully annuitized case, we follow Parpas and Webster (2013) and reformulate this stochastic optimization problem as a deterministic problem that takes $V(t,W(t),j), j\neq i$, as exogenous. Using the same technique outlined in the proof of **Lemma 1**, it is straightforward to show that the optimal value function for the following auxiliary deterministic optimization problem also satisfies the HJB system of equations given by (18):

$$V(0, W_0, i) = \max_{c_i(t)} \left[\int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j \neq i} \lambda_{ij}(t) V(t, W(t), j) \right) dt \right] \tag{19}$$

$$\text{s.t.} \frac{\partial W(t)}{\partial t} = rW(t) + m_i(t) - c_i(t)$$

where, as before, $V(t, W(t), j)$ is taken as given. The Hamiltonian corresponding to (19) is

$$H\left(W(t), c_i(t), p_t^{(i)}\right) = e^{-\rho t} \tilde{S}(i,t) \left(u\left(c_i(t), q_i(t)\right) + \sum_{j \neq i} \lambda_{ij}(t) V(t, W(t), j) \right) + p_t^{(i)} [rW(t) - c_i(t) + m_i(t)]$$

The adjoint equation is

$$\dot{p}_t^{(i)} = -p_t^{(i)} r - e^{-\rho t} \tilde{S}(i,t) \sum_{j \neq i} \lambda_{ij}(t) \frac{\partial V(t, W(t), j)}{\partial W(t)}$$

The solution is obtained by variation of the constant:

$$p_t^{(i)} = \left[\int_t^T e^{(r-\rho)s} \tilde{S}(i,s) \sum_{j \neq i} \lambda_{ij}(s) \frac{\partial V(s, W(s), j)}{\partial W(s)} ds \right] e^{-rt} + \theta^{(i)} e^{-rt}$$

The first order condition for consumption is:

$$p_t^{(i)} = e^{-\rho t} \widetilde{S}(i,t) u_c\left(c_i(t), q_i(t)\right) \tag{20}$$

where the marginal utility of wealth at time $t = 0$ is $\frac{\partial V(0, \overline{W}_0, i)}{\partial \overline{W}(0,i)} = p_0^{(i)} = u_c\left(c_i(0), q_i(0)\right)$.

To analyze the value of life, we again let $\delta(t)$ be a perturbation on the mortality rate with $\int_0^T \delta(t) dt = 1$.

Lemma 5:

The marginal utility of life extension is equal to:

$$\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T \left[e^{-\rho t} \left(\int_0^t \delta(s) ds \right) \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j \neq i} \lambda_{ij}(t)\, V(t, W(t), j) \right) \right] dt$$

Proof of Lemma 5:

From (19), the marginal utility of life extension is defined as

$$\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} = \frac{\partial}{\partial \varepsilon} \int_0^T e^{-\rho t} \exp\left\{ -\int_0^t \left(\mu(s) - \varepsilon\delta(s)\right) + \sum_{j \neq i} \lambda_{ij}(s)\, ds \right\} \left(u\left(c_i^\varepsilon(t), q_i(t)\right) \right.$$
$$\left. \left. + \sum_{j \neq i} \lambda_{ij}(t) V\left(t, W^\varepsilon(t), j\right) \right) dt \right|_{\varepsilon=0}$$

$$= \int_0^T e^{-\rho t}\left(\int_0^t \delta(s)ds\right)\tilde{S}(i,t)\left(u(c_i(t),q_i(t)) + \sum_{j\neq i}\lambda_{ij}(t)V(t,W(t),j)\right)dt$$

$$+ \int_0^T e^{-\rho t}\tilde{S}(i,t)\left(u_c\left(c_i^\varepsilon(t),q_i(t)\right)\frac{\partial c_i^\varepsilon(t)}{\partial\varepsilon} + \sum_{j\neq i}\lambda_{ij}(t)\frac{\partial V(t,W(t),j)}{\partial W}\frac{\partial W^\varepsilon(t)}{\partial\varepsilon}\right)dt$$

where $c_i^\varepsilon(t)$ and $W^\varepsilon(t)$ represent the equilibrium variation in $c_i(t)$ and $W(t)$ caused by this perturbation. We conclude the proof by showing that the second term in the last equality is equal to 0. Note that along this path, wealth at time t is equal to

$$W(t) = W_0 e^{rt} + \int_0^t e^{r(t-s)}m_i(s)ds - \int_0^t e^{r(t-s)}c_i(s)ds,$$

which implies $\frac{\partial W^\varepsilon(t)}{\partial\varepsilon} = -\int_0^t e^{r(t-s)}\frac{\partial c_i^\varepsilon(s)}{\partial\varepsilon}ds$. From the solution to the adjoint equation, we know that

$$e^{-\rho t}\tilde{S}(i,t)u_c\big(c_i(t),q_i(t)\big) = \left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j\neq i}\lambda_{ij}(s)\frac{\partial V(s,W(s),j)}{\partial W(s)}ds\right]e^{-rt} + \theta^{(i)}e^{-rt}.$$

Thus we can rewrite the second term in the expression for $\left.\frac{\partial\mathbb{E}U}{\partial\varepsilon}\right|_{\varepsilon=0}$ above as

$$\int_0^T\left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j\neq i}\lambda_{ij}(s)\frac{\partial V(s,W(s),j)}{\partial W(s)}ds + \theta\right]e^{-rt}\frac{\partial c_i^\varepsilon(t)}{\partial\varepsilon}dt$$

$$-\int_0^T e^{-\rho t}\tilde{S}(i,t)\sum_{j\neq i}\lambda_{ij}(t)\frac{\partial V(t,W(t),j)}{\partial W}\int_0^t e^{r(t-s)}\frac{\partial c_i^\varepsilon(s)}{\partial\varepsilon}ds\,dt$$

$$= \int_0^T\left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j\neq i}\lambda_{ij}(s)\frac{\partial V(s,W(s),j)}{\partial W(s)}ds\right]e^{-rt}\frac{\partial c_i^\varepsilon(t)}{\partial\varepsilon}dt$$

$$-\int_0^T\left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j\neq i}\lambda_{ij}(s)\frac{\partial V(s,W(s),j)}{\partial W(s)}ds\right]e^{-rt}\frac{\partial c_i^\varepsilon(t)}{\partial\varepsilon}dt$$

$$+\int_0^T\theta e^{-rt}\frac{\partial c_i^\varepsilon(t)}{\partial\varepsilon}dt$$

$$= \theta\frac{\partial}{\partial\varepsilon}\underbrace{\int_0^T e^{-rt}c_i^\varepsilon(t)dt}_{W_0+\int_0^T e^{-rt}m_i(t)dt}$$

$$= 0$$

QED

Our next result demonstrates that value of statistical life takes the same basic form as in the deterministic case.

Proposition 6:

Choosing once again the Delta-Dirac function for $\delta(\cdot)$ yields

$$\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T \left[e^{-\rho t}\tilde{S}(i,t)\left(u\big(c_i(t),q_i(t)\big) + \sum_{j\neq i} \lambda_{ij}(t)\, V(t,W(t),j) \right)\right] dt$$
$$= \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) u\big(c_{y_t}(t), q_{y_t}(t)\big) dt \,\middle|\, Y_0 = i \right]$$

Dividing the result by the marginal utility of wealth at time $t = 0$ shows that the value of statistical life takes the same basic form as in the deterministic case:

$$VSL_i = \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) \frac{u\big(c_{y_t}(t), q_{y_t}(t)\big)}{u\big(c_{Y_0}(0), q_{Y_0}(0)\big)} dt \,\middle|\, Y_0 = i \right] = \int_0^T e^{-rt} v_i(t) dt \tag{21}$$

where the value of a life-year is equal to the expected utility of consumption normalized by the expected marginal utility of consumption:

$$v_i(t) = \frac{\mathbb{E}\left[S(t) u\left(c_{y_t}(t), q_{y_t}(t)\right) \middle| Y_0 = i \right]}{\mathbb{E}\left[S(t) u_c\left(c_{y_t}(t), q_{y_t}(t)\right) \middle| Y_0 = i \right]}$$

Proof of Proposition 6: see appendix

As before, the value of statistical life is proportional to the expected discounted (lifetime) utility of consumption, and inversely proportional to the marginal utility of consumption. As we shall show below, a negative health shock increases consumption, causing the net effect on VSL to be ambiguous. This parallels the result we showed previously that removing access to annuitization, thereby exposing a consumer to mortality risk, has an ambiguous effect on VSL.

Because the consumer is not insured against mortality risk, consumption will generally jump following the transition to a new health state. The sign of the jump can be positive or negative, depending on the characteristics of the new health state relative to the old state. Transitioning to a state where the current mortality and future expected mortality are high will generally shift consumption forward (see Figure 2), and vice versa. Our next result proves this formally for a two-state case.[9]

Proposition 7:

Let there be $n = 2$ states. Assume that $\overline{\mu}_1(s) < \overline{\mu}_2(s) \forall s$, so that state 1 is "healthy" and state 2 is "sick." Suppose that the consumer transitions from state 1 to state 2 at time t. Then $c_1(t) < c_2(t)$.

Proof of Proposition 7: See appendix.

We can derive an expression for the life-cycle profile of consumption from (20), the first-order condition for p_t. Differentiating with respect to t, plugging in the result for the adjoint equation and its solution, and rearranging yields

[9] The proof can be extended to allow for a larger number of states, but the conditions required to sign the jump in consumption then become a complicated function of the matrix of transition probabilities and state-specific mortality rates. The two-state case conveys the basic result without a meaningful loss of generality.

$$\frac{\dot{c}_i}{c_i} = \sigma(r-\rho) + \sigma\eta\frac{\dot{q}}{q} - \sigma\overline{\mu}_i(t) - \sigma\sum_{j\neq i}\lambda_{ij}(t)\left[1 - \frac{u_c\left(c_j(t), q_j(t)\right)}{u_c\left(c_i(t), q_i(t)\right)}\right] \tag{22}$$

As in the deterministic case, the rate of change is a declining function of the individual's current mortality rate, $\overline{\mu}_i(t)$: removing the annuity market "pulls consumption earlier" in the life-cycle. Unlike in the deterministic case, there is now an additional source of mortality risk, captured by the fourth term in equation (22). This term represents the possibility that the consumer might transition to a different health state in the future, and shifts consumption further still if the consumer is likely to fall ill in the future.

We caution that equation (22) is specific to an individual's health state i and is not directly comparable to equation (8). That is, one cannot infer from equation (22) whether stochastic mortality *on average* causes consumption to shift forward relative to deterministic mortality. In general, one should expect stochastic mortality to shift consumption forward by less than in the deterministic case. Intuitively, this is because a stochastic environment allows an individual react to unanticipated health shocks by adjusting her consumption. Put differently, a deterministic model is equivalent to a stochastic model where the consumer is forced to keep consumption constant across states. Consumers prefer the ability to adjust consumption, so that they can consume less in healthy states and more in sick states. Our numerical exercises, which assume CRRA utility, find that on net stochastic mortality causes consumers to shift consumption forward a bit less than deterministic mortality.

To analyze the value of prevention, let $\delta_{ij}(t)$ be a perturbation on $\lambda_{ij}(t)$, where $\sum_{j\neq i}\int_0^T \delta_{ij}(t)dt = 1$.

Proposition 8:

The marginal utility of preventing an illness is given by:

$$\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T e^{-\rho t}\tilde{S}(i,t)\left[\int_0^t \sum_{j\neq i}\delta_{ij}(s)\,ds\left(u(c_i(t), q_i(t)) + \sum_{j\neq i}\lambda_{ij}(t)\,V(t, W(t), j)\right) - \sum_{j\neq i}\delta_{ij}(t)\,V(t, W(t), j)\right]dt$$

Proof of Proposition 8:

From (19), the marginal utility of preventing an illness is defined as

$$\begin{aligned}\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} &= \left.\frac{\partial}{\partial \varepsilon}\int_0^T e^{-\rho t}\exp\left\{-\int_0^t \bar{\mu}_i(s) + \sum_{j\neq i}\left(\lambda_{ij}(s) - \varepsilon\delta_{ij}(s)\right)ds\right\}\left(u\left(c_i^\varepsilon(t), q_i(t)\right) + \sum_{j\neq i}\left(\lambda_{ij}(t) - \varepsilon\delta_{ij}(t)\right)V(t, W^\varepsilon(t), j)\right)dt\right|_{\varepsilon=0} \\ &= \int_0^T e^{-\rho t}\tilde{S}(i,t)\left[\int_0^t \sum_{j\neq i}\delta_{ij}(s)\,ds\left(u(c_i(t), q_i(t)) + \sum_{j\neq i}\lambda_{ij}(t)\,V(t, W(t), j)\right) - \sum_{j\neq i}\delta_{ij}(t)\,V(t, W(t), j)\right]dt \\ &\quad + \int_0^T e^{-\rho t}\tilde{S}(i,t)\left(u_c\left(c_i^\varepsilon(t), q_i(t)\right)\frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} + \sum_{j\neq i}\lambda_{ij}(t)\frac{\partial V(t, W(t), j)}{\partial W}\frac{\partial W^\varepsilon(t,i)}{\partial \varepsilon}\right)dt\end{aligned}$$

Following the same argument as in the VSL case, the second term in the last equality is equal to 0.

QED

Choosing the Delta-Dirac function for disease j as done previously then yields the expression for the value of statistical illness:

$$VSI = \frac{V(0, W(0), i) - V(0, W(0), j)}{u_c\big(c_i(0), q_i(0)\big)} = VSL(i) - VSL(j)\frac{u_c\left(c_j(0), q_j(0)\right)}{u_c\big(c_i(0), q_i(0)\big)} \tag{23}$$

Removing access to annuity markets breaks the equivalence between treatment and prevention. A consumer necessarily values prevention from the perspective of a healthy state. Thus, VSI is not equal to the simple difference in VSL between the healthy and sick states (as in equation (17)), because VSL in the sick state is valued from the perspective of the sick, who have a lower marginal utility of consumption due to a shorter life span.

It is not apparent from equation (23) whether prevention is more or less valuable than treatment: a reduction in expected survival in the sick state increases VSI, but it can also increase VSL for the reasons we outlined previously. In order to compare prevention to treatment, we turn to numerical exercises.

IV. ESTIMATES OF THE VALUE OF LIFE

IV.A. Framework

We will work with the discrete time analogue of our model and abstract from the role of quality of life, since aggregate, nationally representative data on quality-of-life trends are not generally available. There are n health states. Denote the transition probabilities between health states by:

$$p_{ij}(t) = \mathbb{P}[Y_{t+1} = j | Y_t = i]$$

As in the continuous time model, the mortality rate at time t depends on the individual's health state:

$$q_t = \sum_{j=1}^{n} \bar{q}_t^j \mathbf{1}\{Y_t = j\}$$

where $\left\{\bar{q}_t^j\right\}$ are given and $\mathbf{1}\{Y_t = j\}$ is an indicator variable equal to 1 if the individual is in state j at time t and 0 otherwise. The probability of surviving from time period t to time period s is denoted as $S_t(s)$, where

$$S_t(t) = 1,$$

$$S_t(s) = S_t(s-1)(1 - q_{s-1}), s > t$$

Let c_t be consumption in period t, w_t (non-annuitized) wealth, ρ the utility discount rate, and r the interest rate. Assume that in each period the consumer receives an exogenously determined income, m_t, and that the maximum lifespan of a consumer is T (i.e., $q_T = 1$).[10] Our baseline model assumes there is no bequest motive, although we plan to relax this assumption in a later exercise.

The consumer's maximization problem is

---

[10] Hubbard, Skinner, and Zeldes (1995) show that failing to include a "welfare floor" in the budget constraint causes life-cycle models to overestimate savings for low-income households. Our calibration exercises model median-income individuals, however, for whom this issue is less important.

$$\max_{\{c_t\}} \mathbb{E}_0 \left[\sum_{t=0}^{T} e^{-\rho t} S_0(t) u(c_t) \right]$$

subject to

$$w_0 \text{ given}$$
$$w_t \geq 0$$

$$w_{t+1} = (w_t + m_t - c_t) e^r$$

We assume throughout that $r = \rho = 0.03$ (Siegel 1992), and that utility takes a CRRA form:

$$u(c) = \frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1-\gamma}$$

We have normalized the utility of death at zero. The consumer receives positive utility if she consumes an amount greater than $\underline{c}$, which represents a subsistence level of consumption. Consuming an amount less than $\underline{c}$ generates utility that is worse than death. Although adding a constant to the utility function has no effect in most settings, Rosen (1988) discusses how the level of utility matters when valuing life extension and notes that well-behaved preferences requires that utility be positive. We are unaware of any empirical evidence on the magnitude of $\underline{c}$, the subsistence level of consumption in the United States. We assume it is equal to $5,000.

The parameter γ is the inverse of the elasticity of intertemporal substitution, an important determinant of the value of life and the value of annuitization. We set $\gamma = 2$ in our analyses. As points of reference, Murphy and Topel (2006) argue that γ is approximately equal to 1, but Brown (2001) uses survey data to estimate a mean value of $\gamma = 3.95$.

We employ dynamic programming techniques to solve for the optimal consumption path (see Appendix for details). The value function is defined as:

$$V(t, w_t, i) = \max_{\{c_t\}} \mathbb{E} \left[\sum_{s=t}^{T} e^{-\rho(s-t)} S_t(s) u(c_s) \middle| Y_t = i \right]$$

We can use the value function to rewrite the optimization problem as a recursive Bellman equation:

$$V(t, w_t, i) = \max_{\{c_t\}} \left[u(c_t) + \frac{1-q_t}{e^\rho} \sum_{j=1}^{N} p_{ij}(t) V(t+1, (w_t + m_t - c_t) e^r, j) \right]$$

Once we have solved for the optimal consumption path, we can use the analytical formulas derived in the previous section to calculate the value of life.[11]

IV.B. Annuitization, retirement policy, and the value of life

In this section we explore the link between annuitization and retirement policy. We build up to these results by calculating how the value of statistical life varies over the life-cycle under alternative

[11] In our calculations of the value of life, we assume that the net savings term is zero, i.e., that savings equals consumption.

annuitization policies. We then calculate how these alternative policies influence the value of life-extension.

We begin the model at age 20 and assume nobody survives past age 100. We use data on age-specific mortality rates from www.mortality.org. Because these mortality data are not available by health state, in this section we will work with a deterministic model (using the above framework, this corresponds to specifying $n = 1$ health state).

Our baseline scenario makes the realistic assumption that the consumer is partially annuitized by virtue of a retirement pension, but lacks access to financial markets and cannot borrow against her future income. Thus, her consumption is limited by current period income and savings from prior periods. We choose the individual's income profile, $\{m_t\}$, to fit data on life-cycle wages as reported by the Current Population Survey for ages less than 65.[12] Beginning at age 65, we assume the individual receives an annuity equal to \$18,001 annually.[13] The wage path for this scenario is displayed as the dotted line labeled "Income (partial annuity)" in Figure 3.

We then construct two alternative scenarios. In the first, the consumer fully annuitizes at age 20 and enjoys a constant annuity stream, $\overline{m}$, provided by an actuarially fair annuities market. This scenario therefore corresponds to the conventional method for modeling the value of life. The second alternative scenario assumes that the consumer's income, $\{m'_t\}$, falls to zero during her retirement years, with a compensating increase during non-retirement years. The income streams in all three scenarios are related according to the following equation:

$$\sum_{t=1}^{T} \frac{m_t S_t}{(1+r)^{t-1}} = \sum_{t=1}^{T} \frac{m'_t S_t}{(1+r)^{t-1}} = \overline{m} \sum_{t=1}^{T} \frac{S_t}{(1+r)^{t-1}}$$

Our assumed interest rate and our data on mortality and income imply an annuity value of $\overline{m} = \$35{,}563$.

The life-cycle profiles of consumption for the three scenarios are displayed in Figure 3. Because the consumer discount rate is equal to the interest rate, the annuitized individual enjoys constant consumption over time.[14] Consumption for the incompletely annuitized individual, by contrast, is constrained by her low income in early life. She saves during middle age when income is high, and then consumes her savings during retirement until eventually her consumption equals her pension. Her consumption is higher than the annuitized individual's consumption between the ages of 30 and 70. This is attributable to "shifting consumption forward" in response to mortality risk. This effect is larger for the "no annuity" scenario than the "partial annuity" scenario, and is particularly dramatic in the final 10 years of life, when old consumers outlive their wealth. This is not surprising: a primary benefit of an annuity is its ability to provide income to consumers in their oldest ages.

---

[12] These data are available at http://data.bls.gov/pdq/querytool.jsp?survey=le. We smooth the data by fitting it to a quadratic polynomial in age.

[13] The average Social Security benefit for retirees in 2013 was \$15,528. Real median income of the elderly was \$18,001 in 2008 (see http://www.ebri.org/pdf/notespdf/EBRI_Notes_06-June10.Inc-Eld.pdf)

[14] Murphy and Topel (2006) generate an inverse U-shaped profile for consumption for fully annuitized consumers by assuming that (1) $r > \rho$; (2) health and the marginal utility of consumption are complements; and (3) quality of life declines with age. While the third assumption is not controversial, the empirical evidence on the first two assumptions is mixed.

Figure 4 shows that this difference in consumption causes a corresponding difference in the value of a statistical life-year (VSLY). The imperfectly annuitized individuals places a low value on VSLY in early and late ages, when consumption is low. Because the fully annuitized individual enjoys constant consumption, however, she values life-years at a constant rate throughout her life.

Figure 5 displays the corresponding value of statistical life (VSL) for these three scenarios, as calculated by equations (3) and (7). Discounting and future mortality cause VSL to decline monotonically with age for a fully annuitized individual. By contrast, the rising value of VSLY early in life generates an inverted U-shape for partially annuitized individuals. At age 40, VSL is equal to \$4.9 million and \$6.6 million for fully annuitized and partially annuitized individuals, respectively. The value for an individual with no annuitization lies in between. All these values are within the ranges estimated by empirical studies of VSL for working-age individuals (Viscusi and Aldy 2003). These results demonstrate that assuming full annuitization is not an innocuous assumption. For example, the fully annuitized model implies that willingness to pay to extend life is highest at the youngest ages, while the more realistic incompletely annuitized model indicates that willingness to pay peaks around age 40.

Figure 5 shows that VSL is greater at older ages for individuals with retirement annuities than it is for individuals with no annuities. Moreover, the value of life peaks at \$7 million for partially annuitized consumers but only at \$6 million for non-annuitized consumers. This suggests that public annuity programs are complementary with retiree healthcare programs and other investments in life-extension for the older population, but substitutable with similar programs for the young.

Next, we employ equations (1) and (5) to calculate the value of permanent reductions in mortality from different diseases.[15] Figure 6 shows that reducing cancer mortality by 10 percent is worth \$20,000 to a fifty-year-old individual under typical assumptions regarding retirement benefits ("partial annuity"). That value drops by 25 percent if the individual is either fully annuitized or has no annuities at all. Discounting causes the value to be worth much less to the young, and low remaining life expectancy coupled with low wealth causes it to also be worth little to the old.

By contrast, a 10 percent reduction in mortality from homicides is worth the most to the young, especially if they are fully annuitized, and reductions in mortality from infectious diseases remain valuable into old age.

We can calculate the total social value of the mortality reductions shown in Figure 6 by aggregating over the age distribution of the 2010 U.S. population.[16] These results, reported in Table 1, are informative for understanding the interaction between retirement policies and the value of health. For example, consider the introduction of Social Security and other defined benefit plans over the last century. Comparing Column (1) to Column (2) of Table 1 suggests that this has raised the value of a 10 percent cancer mortality reduction by \$450 billion, or 15 percent. Completing the U.S. annuity market would add \$100 billion more to this value. Retirement plans raised the value of a 10 percent reduction in all-cause mortality by \$1.4 trillion.

More generally, the shift towards publicly and privately funded retirement plans has raised the value of life at older ages, and thus may have contributed to the observed increase in demand for elderly

[15] The age-specific contribution of different diseases to mortality is available from www.cdc.gov/nchs/data/dvs/lcwk1_2010.pdf.

[16] Unlike Murphy and Topel (2006), we do not account for the value that mortality reductions generate for future (unborn) populations.

healthcare. This dovetails with the point, made by Philipson and Becker (1998), that the moral hazard effects of retirement programs also increase the willingness to pay for longevity. It is therefore not surprising that public spending on healthcare – particularly for the elderly – has grown enormously in developed countries.

Our model predicts that annuitization raises VSL for the elderly. This should cause them to spend more on healthcare and invest more in healthy behaviors, which in turn should ultimately manifest in increased life expectancy. Philipson and Becker (1998) analyze data from Virga (1996) to show that people with more generous annuities live longer than those with less generous annuities. They interpret this as the effect of endogenous longevity investments, which are encouraged among highly annuitized individuals. An additional explanation is that annuitization increases the value of statistical life, as we emphasize here. These are compatible and consistent interpretations.

IV.C. Heterogeneity in VSL and the value of prevention

The conventional economic theory of life extension conceives of VSL as depending primarily on age and consumption. The general framework with stochastic mortality and incomplete annuitization implies instead a substantial amount of variability in VSL within these categories alone. Individuals who have experienced a recent negative mortality shock have systematically higher VSL, but this VSL premium decays over time. We use real-world data on mortality, mortality shocks, and income to estimate the degree to which VSL varies within the traditional categories, and the factors explaining the variation.

We use data on mortality and mortality shocks from the Future Elderly Model (FEM), a widely published microsimulation model that employs nationally representative data from the Health and Retirement Study (Michaud et al. 2011; Goldman et al. 2005; Lakdawalla, Goldman, and Shang 2005; Goldman et al. 2009; Lakdawalla et al. 2009; Goldman et al. 2013; Michaud et al. 2012; Goldman et al. 2010). The FEM uses real-world risks of disease incidence, and mortality rates by disease state, in order to estimate longevity for people over the age of 50 with different comorbid conditions.[17] This is quite useful for our current purposes, because it provides us with an empirically relevant set of estimates for what mortality risk looks like under different disease states.

We divide the health space within the FEM into 20 states. Each state corresponds to the number (0-3) of impaired activities of daily living (ADL) and the number (0-4) of chronic conditions, for a total of $4 \times 5 = 20$ health states. States are ordered first by number of ADL's and then by number of chronic diseases. So state 1 corresponds to 0 ADL's and 0 chronic conditions, state 2 corresponds to 0 ADL's and 1 chronic condition, and so on. For each health state and age, the FEM estimates the probability of dying, and the probability of transitioning to each of the other health states in the next year. As in the theoretical model, individuals can transition only to higher-numbered states, i.e., $p_{ij}(t) = 0 \forall j < i$. In other words, all ADL's and chronic conditions are permanent.

The FEM model is estimated separately by sex (male or female) and smoking status (smoker or nonsmoker). All results presented in this section are for female non-smokers. We will also be conditioning on income and wealth, and frequently reporting results by age, to eliminate all the variation that the conventional model of VSL would imply. Table 2 presents basic descriptive statistics for the data provided by the FEM model. Life expectancy at age 50 ranges from 35 years for a healthy individual in state 1 to 15 years for an ill individual in state 20. Columns (6)-(8) of Table 2 report the probability that an individual exits their health state after one year, i.e., acquires at least one new ADL or chronic

[17] A description of its methodology is available at healthpolicy.app.box.com/FEMTechdoc.

condition. Health states are relatively persistent, with exit rates never exceeding 30%. State 20 is an absorbing state with an exit rate of 0 percent.

We make two simplifying assumptions that allow us to generate exact, analytical solutions to the stochastic mortality model: we assume income is not survival contingent, and that negative wealth is allowed. These assumptions imply an equivalence between income and wealth, allowing us to ignore income and to work with wealth only.[18] See the appendix for details on how the model is calculated. We set initial wealth equal to $735,170 at age 50, which corresponds to wealth at age 50 as estimated by the deterministic model presented in the prior section. All other assumptions are the same as before.

Figure 7 displays life-cycle profiles of consumption and VSL for an individual who never develops a chronic condition or an ADL. Because she never suffers any health shocks, consumption and VSL decline smoothly over time. VSL at age 50 is equal to $3.5 million and has the usual interpretation: it is the value that 1,000 individuals would collectively be willing to pay in order to reduce their individual risk of death by 1/1,000.

Figure 8 demonstrates a key mechanism for variability in VSL: The arrival of a health shock can increase VSL, sometimes substantially. The figure displays contrasting plots for an initially healthy individual who develops one ADL at age 60, and then a second ADL plus two chronic conditions at age 80. The first shock reduces her life expectancy by 4.1 years. The second one reduces her life expectancy by 7.6 years. In contrast to the healthy consumer, the sick consumer's consumption exhibits discontinuous jumps at age 60 and 80 as a result of the negative health shocks. The first shock is not large enough to stave off the declining trend in VSL, but the second one increases VSL at age 80 by over 50 percent, from $1 million to $1.5 million.

The arrival of shocks at the individual level in turn generate substantial variability in VSL in the aggregate. Figure 9 shows VSL at age 50 for individuals in one of the twenty different possible health states, as a function of life expectancy in each of those states. It ranges from $3.5 million to $4.5 million. This understates the total amount of heterogeneity that will occur, because shocks after age 50 will cause individuals' consumption paths to diverge even further. For example, at age 51 there are 210 ($= 20 \times 21/2$) different possible values for VSL.

The stochastic mortality approach also allows us to calculate the value of preventing illness, which we defined as the value of a statistical illness (VSI). Figure 10 plots VSI at age 50 from the perspective of a healthy individual. Each point represents the healthy individual's willingness to pay for a marginal reduction in the probability of developing an illness corresponding to one of the 19 other health states. The values are increasing functions of life expectancy in the sick state because it is more valuable to prevent the onset of a lethal disease than a mild one. The highest VSI value is $1.5 million, which corresponds to preventing the onset of a sick state with 3 ADL's and 4 chronic conditions. The interpretation of this value is analogous to VSL: it is the amount that 1,000 individuals would collectively be willing to pay in order to reduce their risk of developing this illness by 1/1000. In our framework, VSL can be interpreted as the willingness to pay to avoid the "illness" of dying, which correspond to a state with 0 years of remaining life expectancy. VSI corresponds to the willingness to pay to avoid a lower life expectancy, but still living, state.

[18] Generalizing the model to allow for partial annuitization, bequest motives, and other similar modifications is possible but prohibits the calculation of an exact solution. We probe the sensitivity of our results to these alternatives with the deterministic mortality model.

How does the value of prevention compare to the value of treatment? We investigate this question by normalizing VSL and VSI by the number of life-year's "saved." We report the results of those calculations in Table 3. We normalize by discounted life expectancy rather than just life expectancy in order to facilitate comparison to the large literature on cost-effectiveness. Our VSL estimate implies that an individual with one chronic condition and no ADL's (health state 2) has a marginal willingness-to-pay of $187,000 per life-year for a treatment that extends her life. By contrast, our VSI estimate implies that a healthy individual (health state 1) is only willing to pay $147,000 per life-year for a preventive treatment that reduces her chances of developing 1 chronic condition. The ratio of these values is equal to 1.28. Column (7) of Table 2 shows that this ratio is always greater than 1, and sometimes significantly so: treatments for very sick individuals with 3 ADL's and 4 chronic conditions are worth 2.6 times more than equally effective preventive care for healthy individuals. This is in stark contrast to the standard cost-effectiveness framework, which values prevention and treatment equally. The model developed earlier provides the intuition behind these differences: all else equal, the marginal utility of consumption is higher for a healthy individual than for someone who has just suffered a health shock that reduces her lifespan. This in turn drives the difference in willingness-to-pay.

V. CONCLUSION

The economic theory surrounding the value of life has been put to many important uses. Yet, like most theories, it suffers from a few anomalies that appear at odds with intuition, common sense, or empirical facts. We have demonstrated that several of these anomalies are easily explained without abandoning the standard framework, simply by relaxing its strong assumptions around the completeness of annuity markets. Moreover, relaxing this assumption generates new predictions with implications for health policy and behavior. In particular, we show that VSL varies with the arrival of mortality shocks and with remaining life expectancy. A given gain in longevity is more valuable to a consumer who has less life remaining, and vice-versa. Even holding wealth and income fixed, VSL may vary by $1 million or more for a 50-year-old. In addition, we demonstrate an interaction between annuity policy and health policy: Completing the annuity market may significantly increase the value of life-extension, especially for the elderly. For instance, the US Social Security program has increased the value of mortality reductions, adding as much as $150 billion to the value of a 1 percent mortality decline.

Our findings have a number of implications for the valuation of health investments and for policy more generally. The value of a statistical life-year will tend to vary across types of risk, not just across types of people. It can be more valuable to add one month of life for a patient facing a highly fatal disease than for one facing a much milder ailment. Thus, health spending should be more targeted towards the severely ill than current economic models of cost-effectiveness suggest.

In addition, public programs that expand the market for annuities might simultaneously boost the demand for life-extending technologies. Intuitively, annuities calm consumer fears about outliving their wealth and thus enable more aggressive investments in life-extension. Viewed differently, our results also show that market failures in annuities affect the value of statistical life, and thus the socially optimal level of health care spending.

Finally, our framework offers a single unified framework for valuing both life-extension and the prevention of illness. This provides a more practical tool for policymakers and decision makers, since many health investments involve preventing the deterioration of health, not a direct and immediate mortality risk.

Our analysis raises a number of important questions for further research. First, how does the value of longevity vary with endogenous demand for quality of life? Elsewhere, we have studied how incomplete health insurance enhances the value of medical technology that improves quality of life, because such technology acts as insurance by compressing the difference in utility between the sick and healthy states (Lakdawalla, Malani, and Reif 2017). Less clear is how demands for the quantity and quality of life interact with financial market incompleteness of various kinds. Second, what does the generalized value of life model mean for the value of different kinds of medical technologies? For instance, the model suggests that short-term survival gains for high-risk diseases are more valuable than previously believed, but very long-term survival gains might actually be less valuable than previously believed. Finally, what are the implications for the empirical literature on the value of statistical life? Empirical analysis has typically proceeded under the assumption that different kinds of mortality risk are all valued the same way, as long as they imply similar changes in the probability of dying (Viscusi and Aldy 2003; Hirth et al. 2000; Mrozek and Taylor 2002). Our framework casts doubt on this assumption and suggests the need for a more nuanced empirical approach. This missing insight may be one reason for the widely disparate estimates in the empirical literature on the value of a statistical life.

VI. REFERENCES

Arthur, W Brian. 1981. 'The economics of risks to life', *The American Economic Review*: 54-64.

Banks, James, Richard Blundell, and Sarah Tanner. 1998. 'Is there a retirement-savings puzzle?', *American Economic Review*: 769-88.

Bertsekas, D.P. 2005. *Dynamic Programming and Optimal Control* (Athena Scientific).

Brown, Jeffrey R. 2001. 'Private pensions, mortality risk, and the decision to annuitize', *Journal of public economics*, 82: 29-62.

Brown, Jeffrey R, Jeffrey R Kling, Sendhil Mullainathan, and Marian V Wrobel. 2008. 'Why Don't People Insure Late-Life Consumption? A Framing Explanation of the Under-Annuitization Puzzle', *The American Economic Review*, 98: 304-09.

Carroll, Christopher D, and Lawrence H Summers. 1991. 'Consumption growth parallels income growth: Some new evidence.' in, *National Saving and Economic Performance* (University of Chicago Press).

Drummond, Michael F., Mark J. Sculpher, George W. Torrance, Bernie J. O'Brien, and Greg L. Stoddart. 2005. *Methods for the Economic Evaluation of Health Care Programmes*.

Fernandez-Villaverde, Jesus, and Dirk Krueger. 2007. 'Consumption over the life cycle: Facts from consumer expenditure survey data', *The Review of Economics and Statistics*, 89: 552-65.

Goldman, Dana P, David Cutler, John W Rowe, Pierre-Carl Michaud, Jeffrey Sullivan, Desi Peneva, and S Jay Olshansky. 2013. 'Substantial health and economic returns from delayed aging may warrant a new focus for medical research', *Health Affairs*, 32: 1698-705.

Goldman, Dana P, Pierre-Carl Michaud, Darius Lakdawalla, Yuhui Zheng, Adam Gailey, and Igor Vaynman. 2010. 'The fiscal consequences of trends in population health', *National Tax Journal*, 63: 307-30.

Goldman, Dana P, Baoping Shang, Jayanta Bhattacharya, Alan M Garber, Michael Hurd, Geoffrey F Joyce, Darius N Lakdawalla, Constantijn Panis, and Paul G Shekelle. 2005. 'Consequences of health trends and medical innovation for the future elderly', *Health Affairs*, 24: W5.

Goldman, Dana P, Yuhui Zheng, Federico Girosi, Pierre-Carl Michaud, S Jay Olshansky, David Cutler, and John W Rowe. 2009. 'The benefits of risk factor prevention in Americans aged 51 years and older', *American Journal of Public Health*, 99: 2096.

Green, C., and K. Gerard. 2009. 'Exploring the social value of health-care interventions: a stated preference discrete choice experiment', *Health Economics*, 18: 951-76.

Hall, Robert E, and Charles I Jones. 2007. 'The Value of Life and the Rise in Health Spending', *The Quarterly Journal of Economics*, 122: 39-72.

Hirth, Richard A, Michael E Chernew, Edward Miller, A Mark Fendrick, and William G Weissert. 2000. 'Willingness to pay for a quality-adjusted life year in search of a standard', *Medical Decision Making*, 20: 332-42.

Hubbard, R Glenn, Jonathan Skinner, and Stephen P Zeldes. 1995. 'Precautionary Saving and Social Insurance', *Journal of Political Economy*, 103.

Lakdawalla, Darius, Anup Malani, and Julian Reif. 2017. 'The Insurance Value of Medical Innovation', *Journal of public economics*, 145: 94-102.

Lakdawalla, Darius N, Dana P Goldman, Pierre-Carl Michaud, Neeraj Sood, Robert Lempert, Ze Cong, Han de Vries, and Italo Gutierrez. 2009. 'US pharmaceutical policy in a global marketplace', *Health Affairs*, 28: w138-w50.

Lakdawalla, Darius N, Dana P Goldman, and Baoping Shang. 2005. 'The health and cost consequences of obesity among the future elderly', *Health Affairs*, 24: W5R30.

Linley, Warren G., and Dyfrig A. Hughes. 2013. 'Societal views on NICE, Cancer Drugs Fund, and Value Based Pricing Criteria for Prioritising Medicines: A Cross-Sectionalk Survey of 4118 Adults in Great Britain', *Health Economics*, 22: 948-64.

Michaud, Pierre-Carl, Dana Goldman, Darius Lakdawalla, Adam Gailey, and Yuhui Zheng. 2011. 'Differences in health between Americans and Western Europeans: Effects on longevity and public finance', *Social Science and Medicine*, 73: 254-63.

Michaud, Pierre-Carl, Dana P Goldman, Darius N Lakdawalla, Yuhui Zheng, and Adam H Gailey. 2012. 'The value of medical and pharmaceutical interventions for reducing obesity', *Journal of Health Economics*, 31: 630-43.

Mrozek, Janusz R, and Laura O Taylor. 2002. 'What determines the value of life? a meta - analysis', *Journal of Policy Analysis and Management*, 21: 253-70.

Murphy, Kevin M, and Robert H Topel. 2006. 'The Value of Health and Longevity', *Journal of Political Economy*, 114: 871-904.

Nord, E., J. Richardson, A. Street, H. Kuhse, and P. Singer. 1995. 'Maximizing health benefits vs egalitarianism: an Australian survey of health issues', *Social Science and Medicine*, 41: 1429-37.

Parpas, Panos, and Mort Webster. 2013. 'A stochastic minimum principle and an adaptive pathwise algorithm for stochastic optimal control', *Automatica*, 49: 1663-71.

Philipson, Tomas J, and Gary S Becker. 1998. 'Old-age longevity and mortality-contingent claims', *Journal of Political Economy*, 106: 551-73.

Reichling, Felix, and Kent Smetters. 2015. 'Optimal Annuitization with Stochastic Mortality and Correlated Medical Costs', *The American Economic Review*, 105: 3273-320.

Rosen, Sherwin. 1988. 'The value of changes in life expectancy', *Journal of Risk and Uncertainty*, 1: 285-304.

Shepard, Donald S, and Richard J Zeckhauser. 1984. 'Survival versus consumption', *Management Science*, 30: 423-39.

Siegel, Jeremy J. 1992. 'The real rate of interest from 1800–1990: A study of the US and the UK', *Journal of Monetary Economics*, 29: 227-52.

Virga, M. 1996. 'Earn More, Live Longer-Variation in Mortality by Income Level', *Pension Section News*: 1-7.

Viscusi, W Kip, and Joseph E Aldy. 2003. 'The value of a statistical life: a critical review of market estimates throughout the world', *Journal of Risk and Uncertainty*, 27: 5-76.

Yaari, Menahem E. 1965. 'Uncertain lifetime, life insurance, and the theory of the consumer', *The review of economic studies*: 137-50.

VII. TABLES AND FIGURES

Table 1. Aggregate value (in billions of 2010 dollars) of a permanent 10 percent reduction in mortality from selected diseases

	(1)	(2)	(3)
Cause of death	**No annuity**	**Partial annuity**	**Full annuity**
All causes	$9,978	$11,422	$12,343
Cancer	$2,901	$3,346	$3,446
Diabetes	$318	$366	$382
Heart disease	$2,073	$2,401	$2,535
Homicide	$98	$97	$154
Infectious diseases	$140	$162	$175

Notes: Aggregate values were calculated by using the 2010 U.S. population by age. Column (1) presents estimates under the assumption that individuals have no annuities in retirement. Column (2) presents estimates under the assumption that individuals enjoy standard retirement benefits such as Social Security. Column (3) presents estimates under the assumption that individuals have fully annuitized all wealth and future income at age 20. The net present value of individuals' wealth at age 20 is the same across all three columns.

Table 2. Descriptive statistics for the twenty health states used in the stochastic mortality model

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			Life expectancy (years)			**Exit probability**		
Health state	**ADL's**	**Chronic conditions**	**Age 50**	**Age 70**	**Age 90**	**Age 50**	**Age 70**	**Age 90**
1	0	0	35.0	20.2	8.6	4.5%	9.0%	16.6%
2	0	1	31.5	17.8	7.2	4.9%	9.2%	16.2%
3	0	2	27.5	15.4	5.9	6.3%	9.9%	15.2%
4	0	3	23.3	12.8	4.8	9.5%	11.3%	13.5%
5	0	4	20.2	10.5	3.5	6.9%	9.4%	12.6%
6	1	0	29.7	17.2	7.3	9.6%	13.4%	18.3%
7	1	1	26.6	15.0	5.9	9.4%	13.6%	19.2%
8	1	2	23.0	12.7	4.7	12.6%	14.8%	17.4%
9	1	3	20.2	10.7	3.6	11.4%	13.9%	17.4%
10	1	4	17.3	9.0	3.0	18.3%	15.0%	12.1%
11	2	0	27.5	15.2	5.9	10.4%	17.2%	26.4%
12	2	1	24.1	13.2	4.9	14.1%	18.4%	23.7%
13	2	2	21.7	11.8	4.2	12.9%	15.1%	17.6%
14	2	3	19.0	10.3	3.6	14.8%	14.2%	14.1%
15	2	4	16.8	8.5	2.7	11.7%	11.9%	12.1%
16	3	0	26.6	14.2	5.1	4.8%	8.7%	14.4%
17	3	1	22.5	12.0	4.1	8.1%	10.1%	12.4%
18	3	2	19.5	10.1	3.3	8.5%	9.0%	9.4%
19	3	3	16.9	8.4	2.6	7.8%	7.1%	6.4%
20	3	4	15.2	7.2	2.1	0.0%	0.0%	0.0%

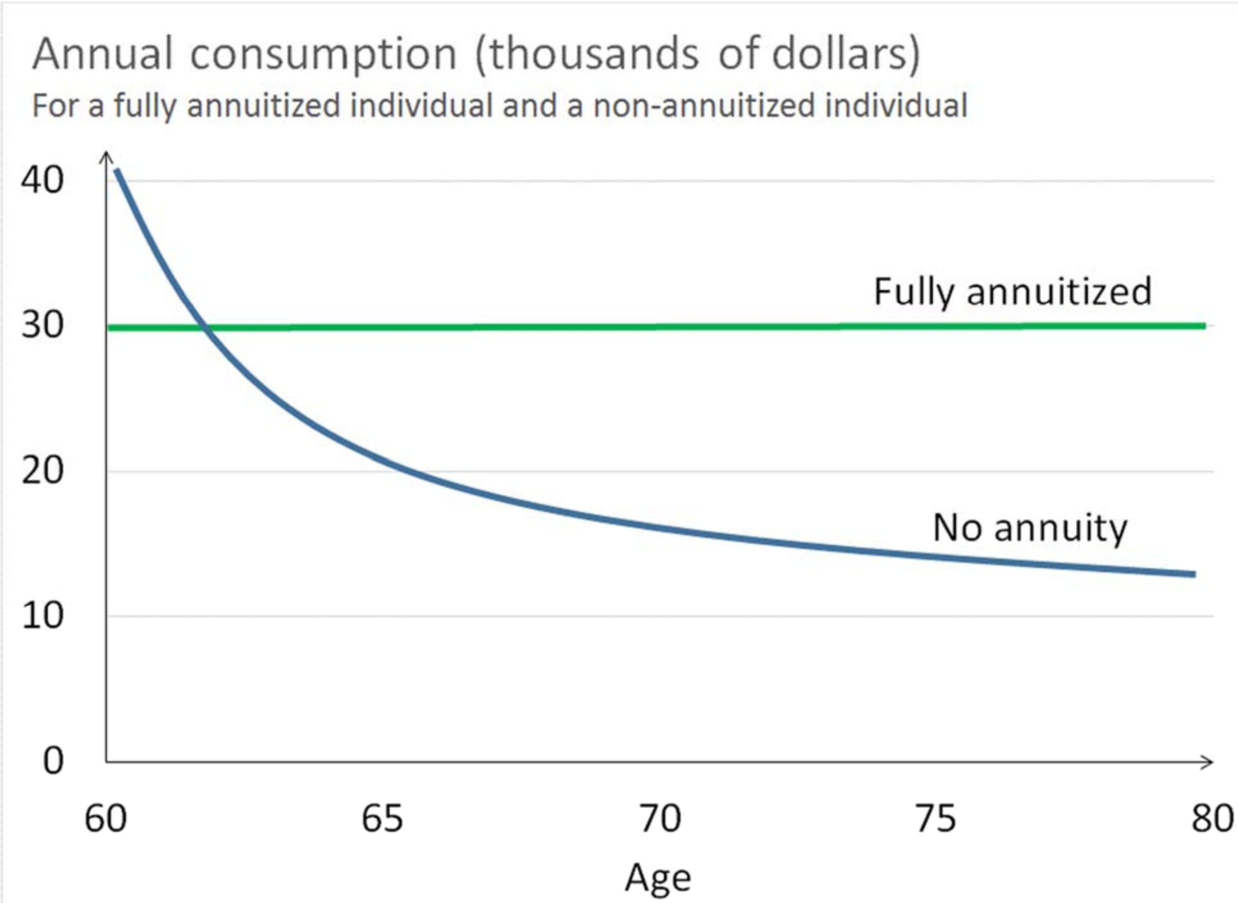
Notes: Columns (1) and (2) report the number of impaired activities of daily living (ADL) and the number of chronic conditions corresponding to the health state. Column (3)-(5) reports the life expectancy for an individual in that health state. Columns (6)-(8) report the probability that an individual transitions to a different health state in the following year. Data source: Future Elderly Model.

Table 3. Value of treatment and prevention (in thousands of dollars) for a 50-year-old

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
					WTP per discounted life-year		
Health state	Life expectancy	Disc. life expectancy	VSL	VSI	Treatment	Prevention	Treatment/ Prevention
1	35.0	20.9	$3,524	N/A	$169	N/A	N/A
2	31.5	19.5	$3,650	$201	$187	$147	1.28
3	27.5	17.8	$3,817	$458	$215	$148	1.45
4	23.3	15.7	$4,019	$769	$256	$150	1.71
5	20.2	14.1	$4,164	$1,025	$296	$151	1.95
6	29.7	18.8	$3,729	$304	$199	$147	1.36
7	26.6	17.4	$3,866	$518	$223	$148	1.50
8	23.0	15.6	$4,036	$789	$259	$150	1.73
9	20.2	14.1	$4,185	$1,026	$297	$151	1.97
10	17.3	12.4	$4,354	$1,286	$351	$153	2.30
11	27.5	17.8	$3,813	$458	$215	$148	1.45
12	24.1	16.1	$3,973	$706	$246	$150	1.65
13	21.7	14.9	$4,099	$897	$275	$151	1.83
14	19.0	13.5	$4,259	$1,118	$316	$152	2.09
15	16.8	12.2	$4,375	$1,329	$360	$153	2.35
16	26.6	17.3	$3,817	$536	$221	$150	1.47
17	22.5	15.3	$4,025	$843	$264	$151	1.75
18	19.5	13.7	$4,192	$1,093	$307	$152	2.02
19	16.9	12.2	$4,345	$1,327	$357	$153	2.33
20	15.2	11.1	$4,446	$1,496	$400	$154	2.60

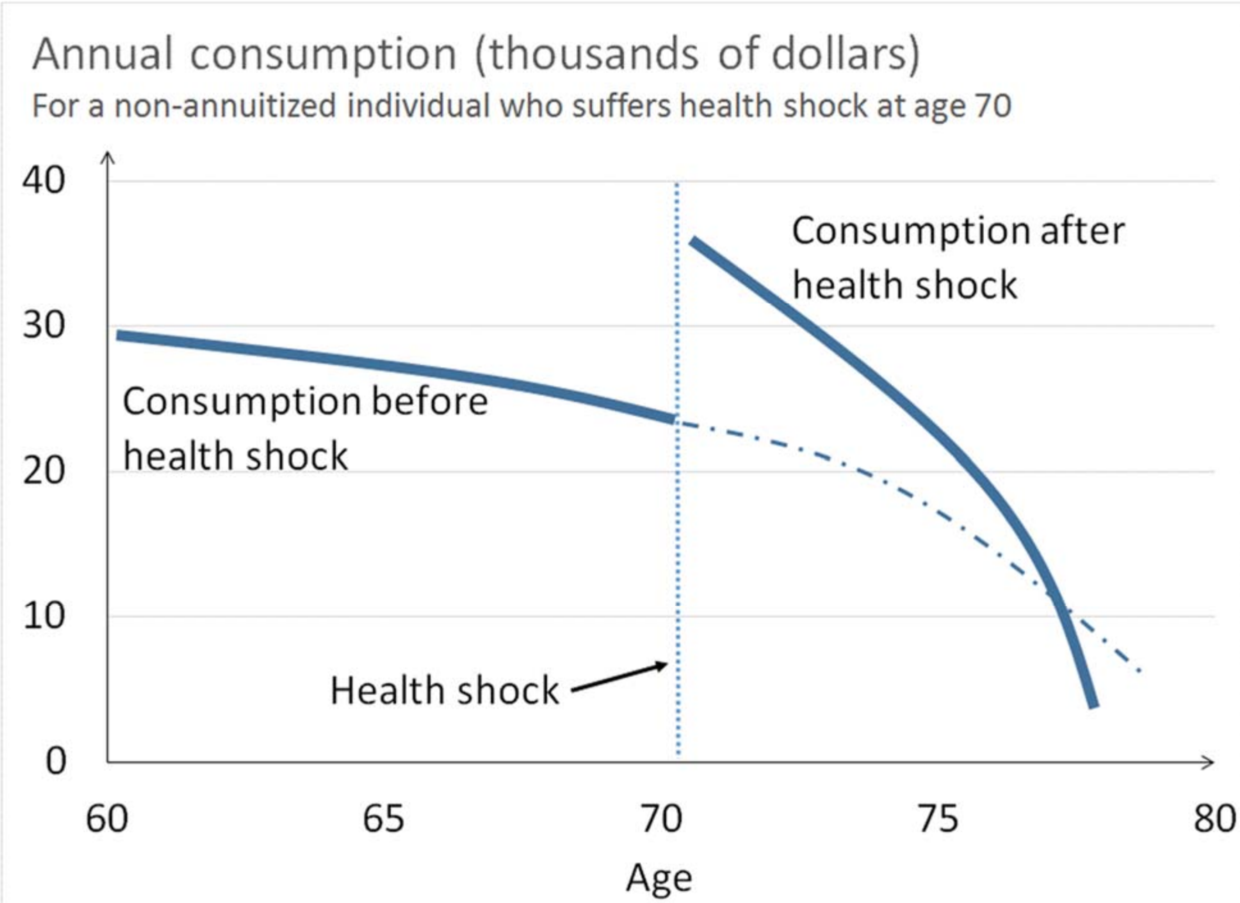
Notes: Table displays values (in thousands of dollars) from a life-cycle modeling exercise where mortality is stochastic. Column (1) reports remaining life expectancy for a 50-year-old for twenty different possible health states. Column (2) reports remaining life expectancy discounted at 3 percent. Column (3) reports value of statistical life (VSL) for an individual in each health state. Column (4) reports value of statistical illness (VSI) for a healthy individual in state 1. Column (5) reports a sick individual's willingness-to-pay (WTP) per discounted life-year for a therapeutic treatment, which is equal to the value in column (3) divided by the value in column (2). Column (6) reports the healthy individual's corresponding WTP for a preventive treatment, which is equal to the value in column (4) divided by (20.9 minus the value in column (2). Column (7) reports the ratio of the values reported in columns (5) and (6).

Figure 1. Illustrative example: annual consumption for fully annuitized and non-annuitized consumers



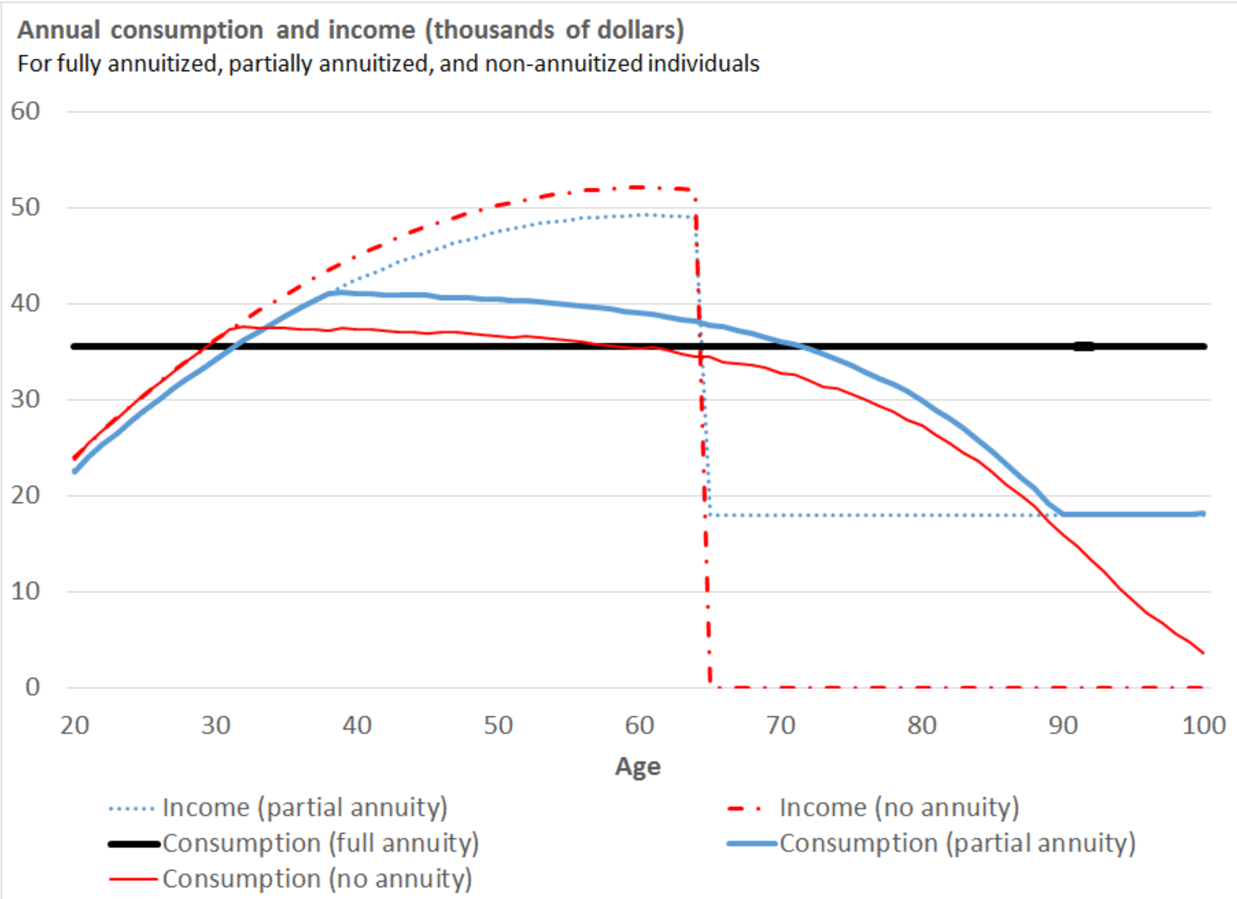
Notes: It is optimal for a non-annuitized consumer who is exposed to mortality risk to shift her consumption forward in time.

Figure 2. Illustrative example: health shocks induce a consumer to increase consumption when she is not annuitized



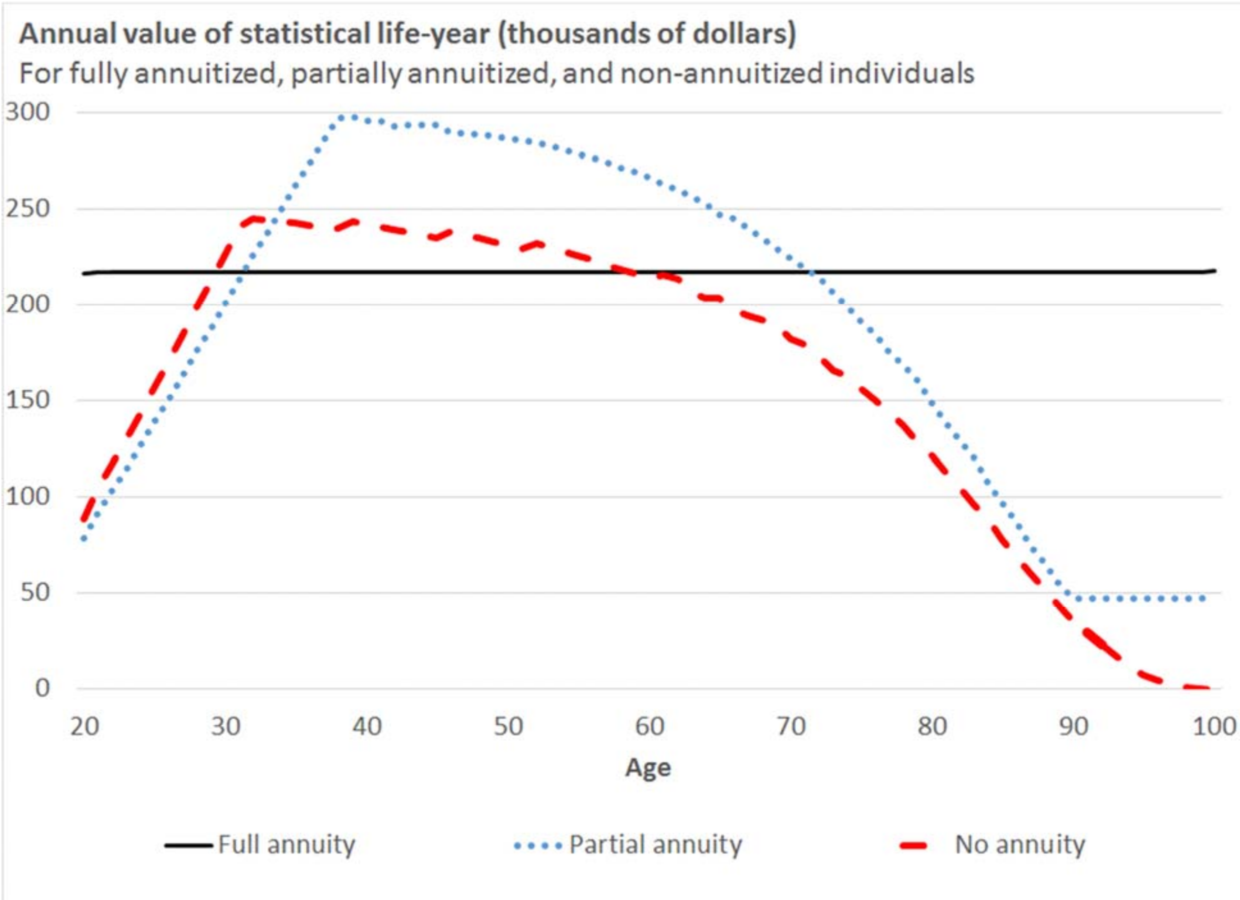
Notes: Consider a hypothetical, non-annuitized individual who suffers a health shock at age 70 that significantly increases her annual mortality. It is optimal for this consumer to "spend down" her wealth by increasing consumption.

Figure 3. Life-cycle profiles of consumption and income



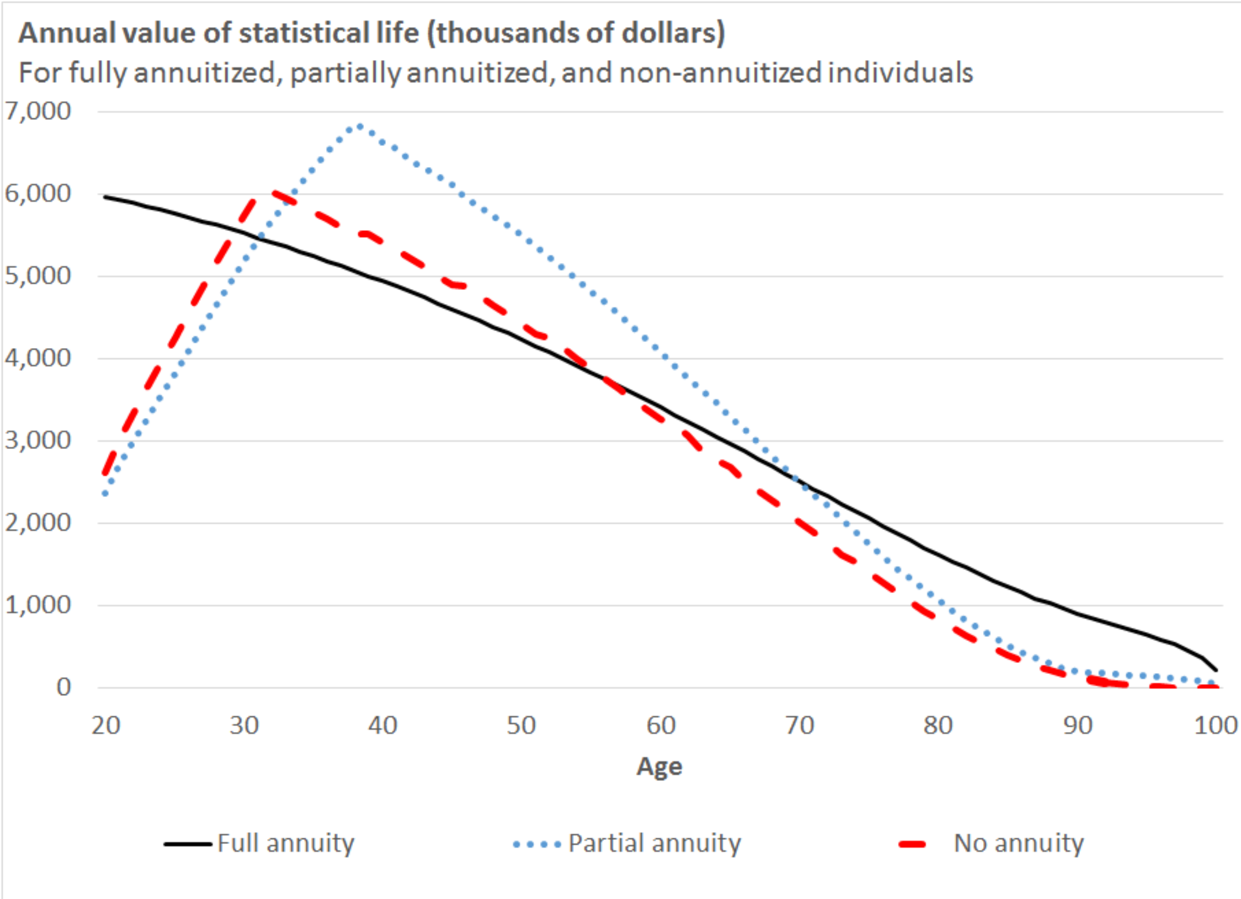
Notes: Figure plots consumption results from a life-cycle modeling exercise where mortality is deterministic. "Full annuity" line displays consumption (= income) when the consumer fully annuitizes all wealth and future earnings at age 20. "Partial annuity" displays consumption for consumer who receives typical social security and retirement benefits. "No annuity" displays consumption for a consumer whose income falls to 0 upon retirement at age 65. The net present value at age 20 of all future income is identical in all three scenarios.

Figure 4. Life-cycle profile of value of statistical life-year



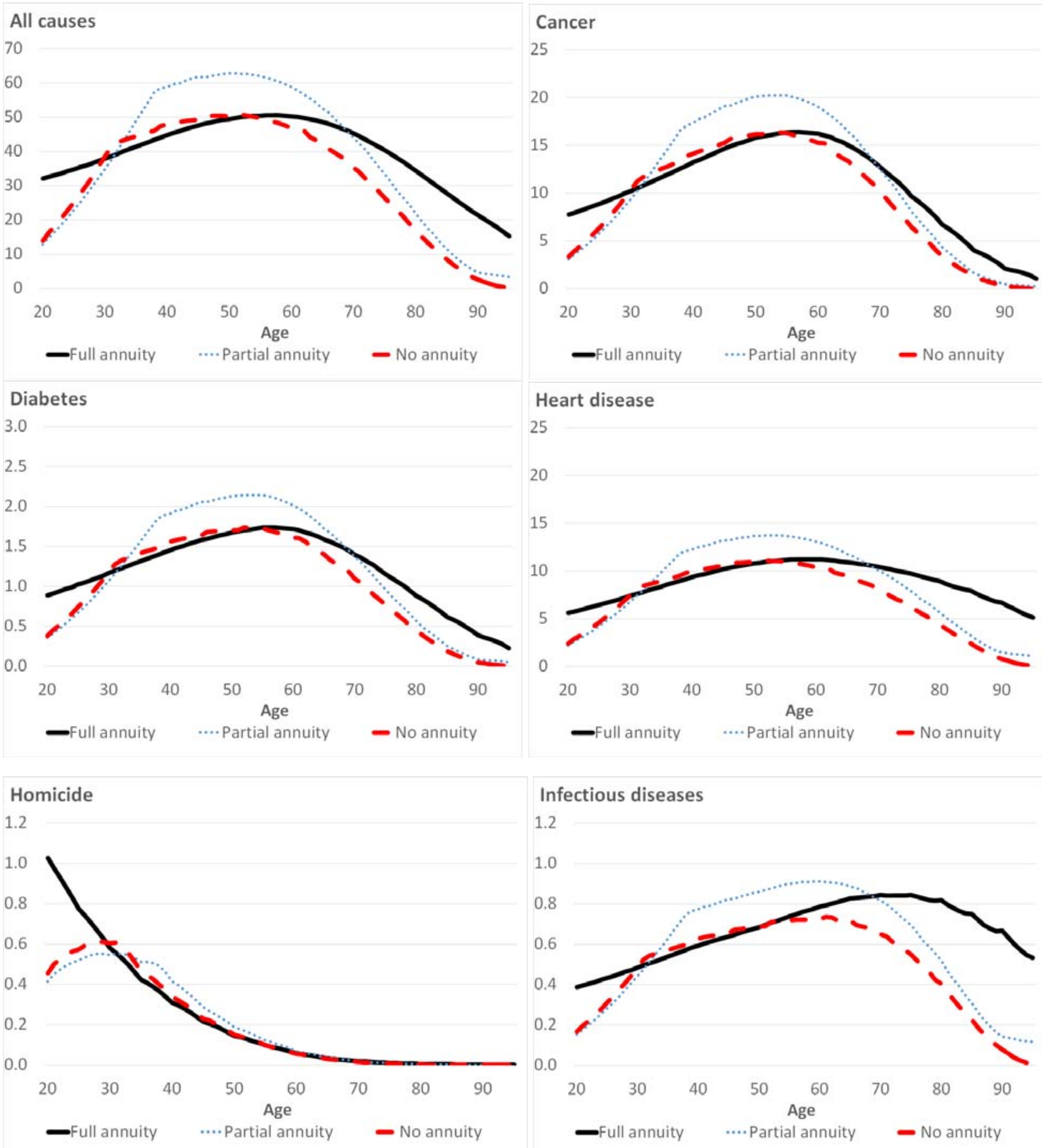
Notes: Figure plots the value of a statistical life-year for the three scenarios displayed in Figure 3. “Full annuity” assumes the consumer fully annuitizes all wealth and future earnings at age 20. “Partial annuity” assumes the consumer receives typical social security and retirement benefits. “No annuity” assumes the consumer’s income falls to 0 upon retirement at age 65. The net present value at age 20 of all future income is identical in all three scenarios.

Figure 5. Life-cycle profile of the value of statistical life when mortality is deterministic



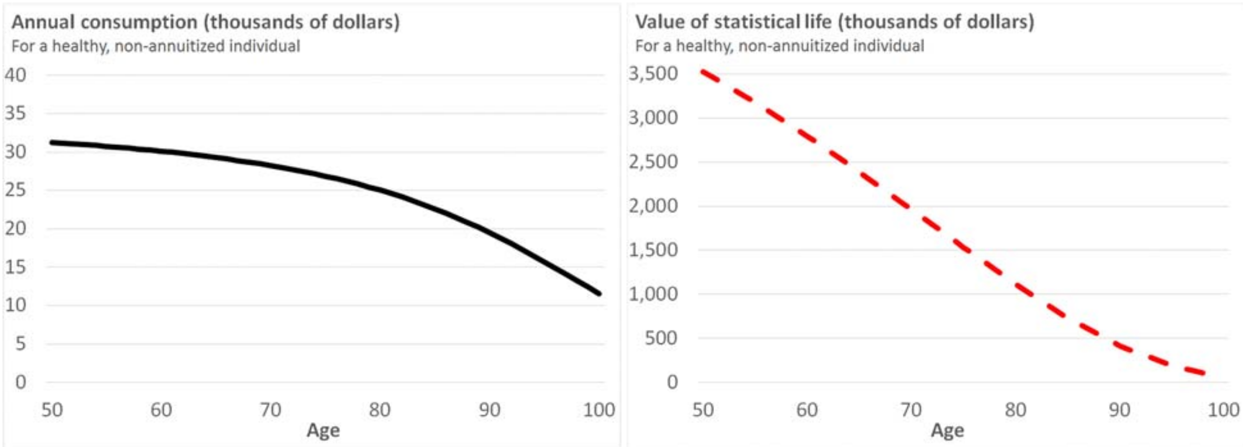
Notes: Figure plots the value of statistical life for the three scenarios displayed in Figure 3. "Full annuity" assumes the consumer fully annuitizes all wealth and future earnings at age 20. "Partial annuity" assumes the consumer receives typical social security and retirement benefits. "No annuity" assumes the consumer's income falls to 0 upon retirement at age 65. The net present value at age 20 of all future income is identical in all three scenarios.

Figure 6. Current value of a 10 percent, permanent reduction in mortality from selected diseases



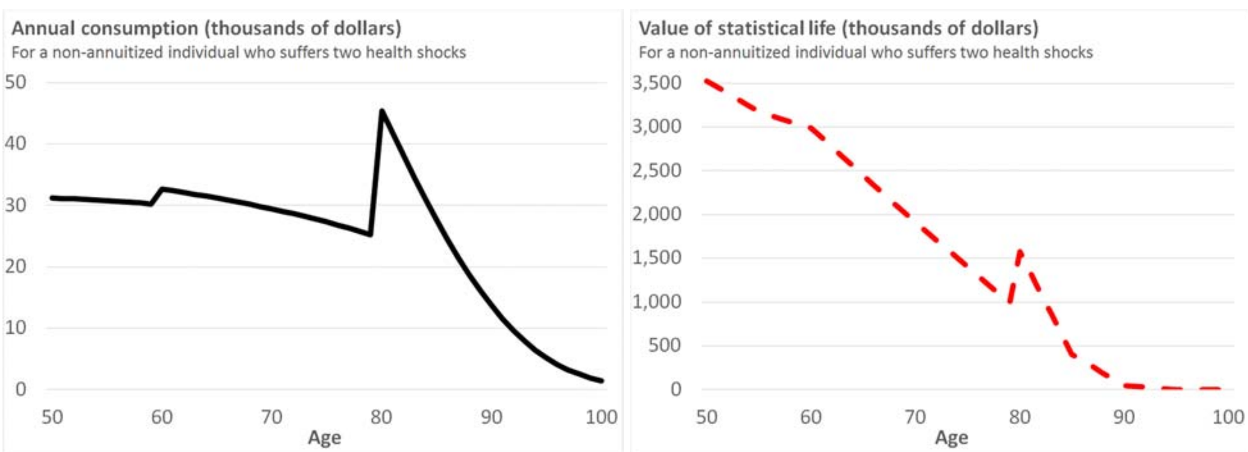
Notes: Figures plot results from a life-cycle modeling exercise where mortality is deterministic. Value of mortality reduction is calculated using equations (1) and (5).

Figure 7. Consumption and value of life when mortality is stochastic and individual is always healthy



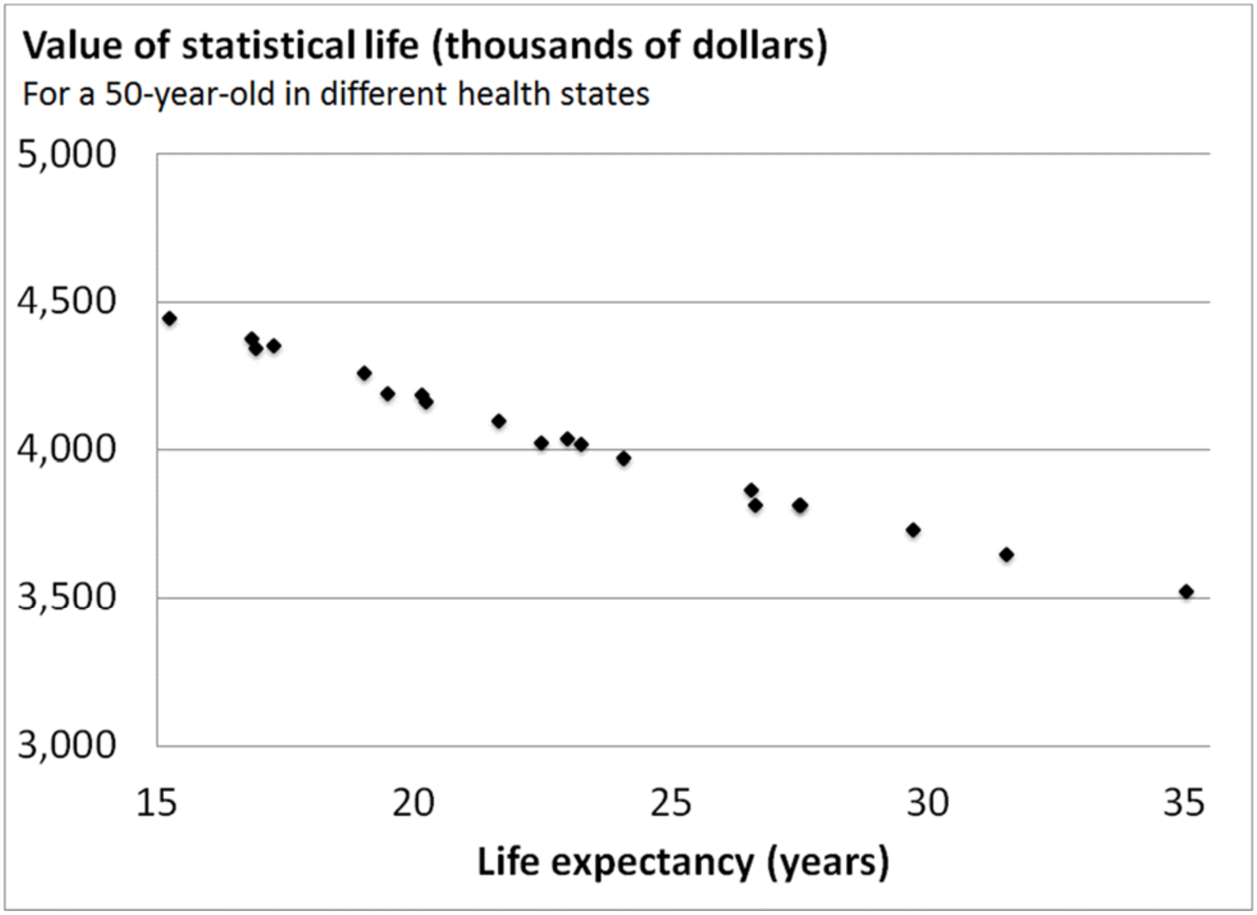
Notes: The left figure plots an individual's consumption profile from a life-cycle modeling exercise where mortality is stochastic and the consumer never falls ill. The right figure plots the corresponding value of statistical life for this individual.

Figure 8. Consumption and value of life when mortality is stochastic and individual falls ill



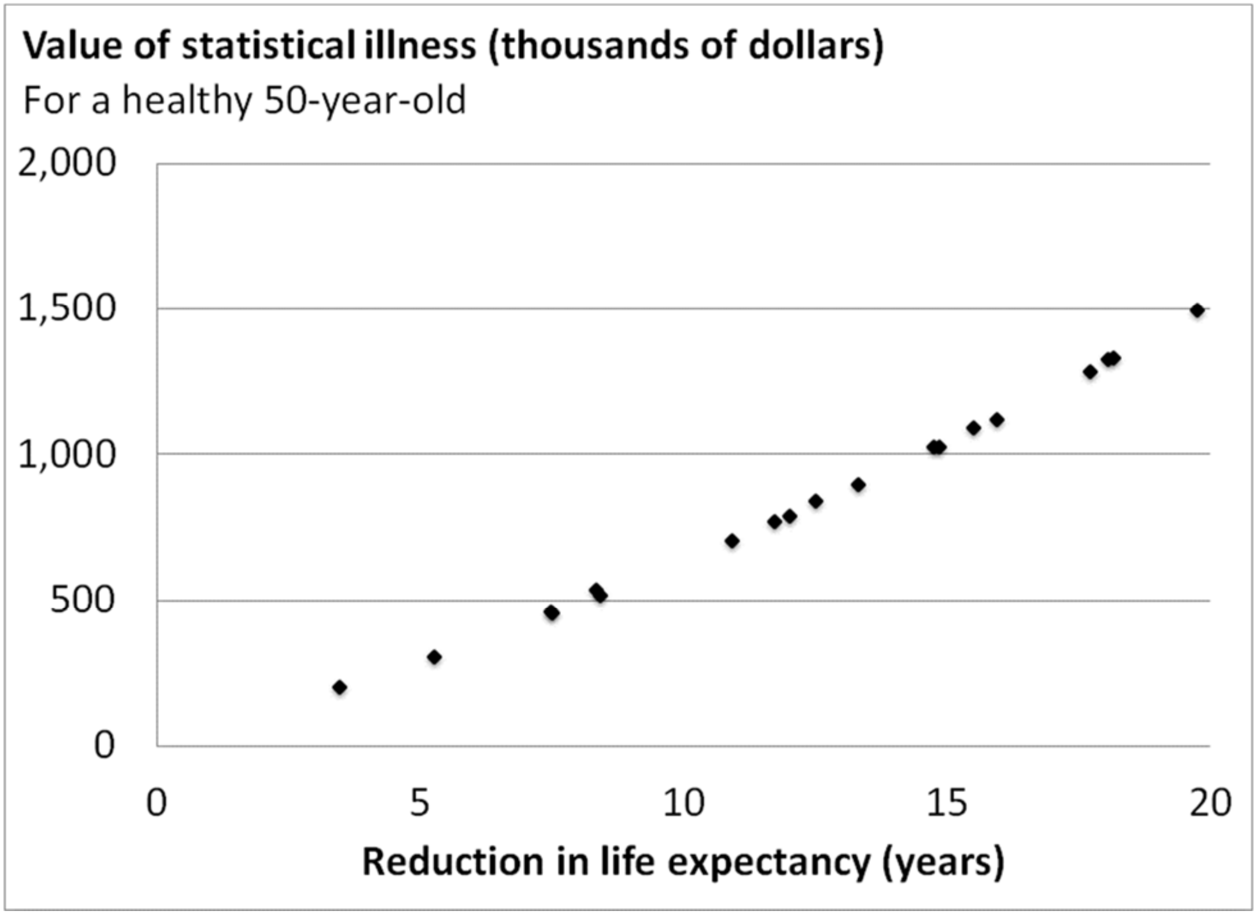
Notes: The left figure plots annual consumption for an individual who falls ill twice in her life. At age 60, this causes her difficulties with one routine activity of daily living (ADL). At age 80, she is diagnosed with two chronic conditions and subsequently has difficulties with two ADL's. The right figure plots the corresponding value of statistical life (VSL) for this individual. The second illness is severe enough that it causes a 50 percent increase in her VSL.

Figure 9. Value of statistical life (VSL) at age 50, as a function of remaining life expectancy



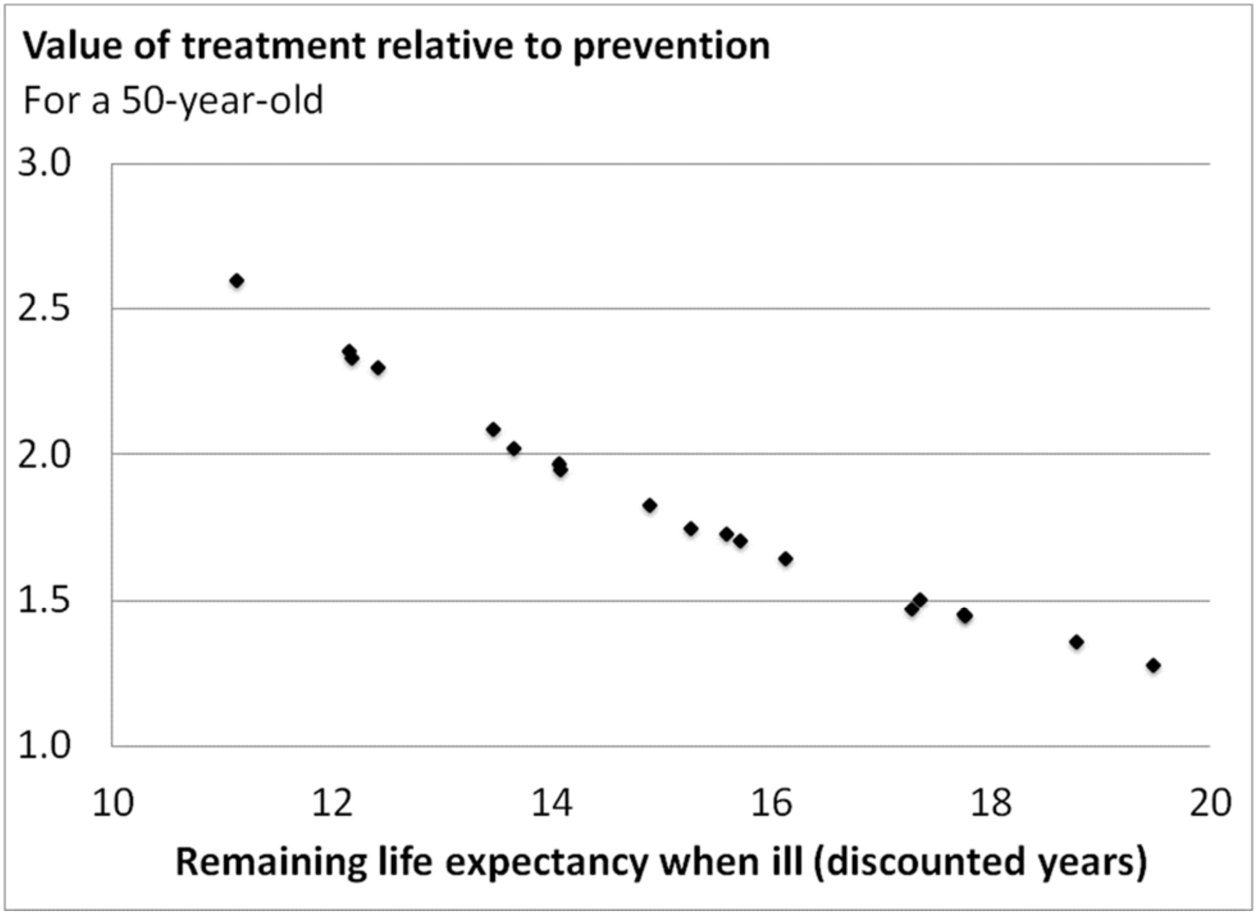
Notes: Figure plots VSL at age 50 for the twenty different health states employed in the stochastic mortality model. The remaining life expectancy in these states ranges from 15.2 years to 35.0 years. VSL ranges from $3.5 million to $4.4 million. These data are also reported in columns (1) and (3) Table 3.

Figure 10. Value of statistical illness (VSI) at age 50, as a function of illness severity



Notes: Figure plots VSI at age 50 for a healthy individual with a remaining life expectancy of 35 years who faces a risk of transitioning to one of 19 alternative, sicker health states. Each point represents the healthy individual's willingness-to-pay for a marginal reduction in the probability of transitioning to a particular sick state. These data are also reported in columns (1) and (4) of Table 3.

Figure 11. Value of treatment relative to prevention at age 50, as a function of (discounted) life expectancy in sick state



Notes: Figure plots the ratio of an individual's willingness-to-pay (WTP) per life-year for preventive care when healthy to the WTP per life-year for a treatment when sick, as a function of life expectancy when sick. Life expectancy is calculated using a discount rate of 3 percent. These data are also reported in columns (2) and (7) of Table 3.

APPENDIX (FOR ONLINE PUBLICATION ONLY)

A. Mathematical proofs

Proof of lemma 1:

Let $V(t, \overline{W}_t, j)$ and $c_j(t), j \neq i$, be taken as given (exogenous). Consider the deterministic optimization problem:

$$V(0, \overline{W}_0, i) = \max_{\{c_i(t)\}} \left\{ \int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j \neq i} \lambda_{ij}(t) V(t, \overline{W}_t, j) \right) dt \right\}$$

subject to

$$\text{s.t.} \frac{\partial \overline{W}(t,j)}{\partial t} = \left(r + \overline{\mu}_j(t) \right) \overline{W}(t,j) - c_j(t) + \sum_{k \neq j} \lambda_{jk}(t) \left[\overline{W}(t,j) - \overline{W}(t,k) \right], j = 1, \dots, n$$

Denote the optimal value-to-go as

$$\tilde{V}(u, \overline{W}_u, i) = \max_{\{c_i(t)\}} \left\{ \int_u^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j \neq i} \lambda_{ij}(t) V(t, \overline{W}_t, j) \right) dt \right\}$$

Setting $\tilde{V}(u, \overline{W}_u, i) = e^{-\rho t} \tilde{S}(i,t) V(t, \overline{W}_t, i)$ then demonstrates that $V(\cdot)$ satisfies the HJB (12) for i.

QED

Proof of Lemma 2:

The adjoint equations for the Hamiltonian (14) are:

$$\dot{p}_t^{(i)} = -p_t^{(i)} \left(r + \overline{\mu}_i(t) + \sum_{j \neq i} \lambda_{ij}(t) \right) + \sum_{l \neq i} \lambda_{li}(t) p_t^{(l)} \text{ and}$$
$$\dot{p}_t^{(k)} = e^{-\rho t} \tilde{S}(i,t) \lambda_{ik}(t) \frac{\partial V(t, \overline{W}_t, k)}{\partial \overline{W}(t,k)} - p_t^{(k)} \left(r + \overline{\mu}_k + \sum_{l \neq k} \lambda_{kl}(t) \right) + \sum_{l \neq k} \lambda_{lk}(t) p_t^{(l)}$$

for $k \neq i$. Suppose that $p_t^{(k)} = 0, k \neq i$. (We will verify this at the end of the proof.) This implies:

$$p_t^{(i)} = \theta e^{-rt} \tilde{S}(i,t)$$

where θ is a constant. Note also that the first-order condition of the Hamiltonian with respect to $c_i(t)$ is

$$e^{-\rho t} \tilde{S}(i,t) u_c(c_i(t), q_i(t)) = p_t^{(i)}$$

Setting these last two equations equal to each other then yields the desired result.

To verify that $p_t^{(k)} = 0$, $k \neq i$, note that the previous result implies via the HJB that $\partial V(t, \overline{W}_t, i) / \partial \overline{W}(t,i) = \theta\, e^{-(r-\rho)t}$, so that the adjoint equation for $k \neq i$ is

$$\dot{p}_t^{(k)} = -\overbrace{\theta e^{-rt}\tilde{S}(i,t)}^{p_t^{(i)}}\lambda_{ik}(t) + p_t^{(i)}\lambda_{ik}(t) - p_t^{(k)}\left(r + \overline{\mu}_k + \sum_{l\neq k}\lambda_{kl}(t)\right) + \sum_{l\neq\{k,i\}}\lambda_{lk}(t)p_t^{(l)}$$

$$= 0$$

QED

Proof of Proposition 4:

Working from equation (13) in the text, the marginal utility of prevention is given by

$$\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} = \frac{\partial}{\partial \varepsilon}\int_0^T e^{-\rho t}\exp\left\{-\int_0^t \overline{\mu}_i(s) + \sum_{j\neq i}\left[\lambda_{ij}(s) - \varepsilon\delta_{ij}(s)\right]ds\right\}\Bigg(u\big(c_i^\varepsilon(t), q_i(t)\big) \\ \left. + \sum_{j\neq i}\left[\lambda_{ij}(t) - \varepsilon\delta_{ij}(t)\right]V\left(t, \overline{W}_t^\varepsilon, j\right)\Bigg)dt\right|_{\varepsilon=0}$$

where $c_i^\varepsilon(t)$ and $\overline{W}_t^\varepsilon$ represent the equilibrium variations in $c_i(t)$ and $\overline{W}_t$ caused by this perturbation. This yields

$$\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T e^{-\rho t}\left(\int_0^t \sum_{j\neq i}\delta_{ij}(s)\,ds\right)\tilde{S}(i,t)\left(u\big(c_i(t), q_i(t)\big) + \sum_{j\neq i}\lambda_{ij}(t)\,V\big(t, \overline{W}_t, j\big)\right) \\ - e^{-\rho t}\tilde{S}(i,t)\sum_{j\neq i}\delta_{ij}(t)\,V\big(t, \overline{W}_t, j\big) \\ + e^{-\rho t}\tilde{S}(i,t)\left(\underbrace{u_c\big(c_i(t), q_i(t)\big)}_{\theta e^{-(r-\rho)t}}\left.\frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon}\right|_{\varepsilon=0} + \sum_{j\neq i}\lambda_{ij}(t)\underbrace{V_W\big(t, \overline{W}_t, j\big)}_{\theta e^{-(r-\rho)t}}\left.\frac{\partial \overline{W}^\varepsilon(t,j)}{\partial \varepsilon}\right|_{\varepsilon=0}\right)dt$$

Next, note that the budget constraint implies

$$0 = \left.\frac{\partial W_0^\varepsilon}{\partial \varepsilon}\right|_{\varepsilon=0} = \frac{\partial}{\partial \varepsilon}\int_0^T e^{-rt}\exp\left\{-\int_0^t \overline{\mu}_i(s) + \sum_{j\neq i}\left[\lambda_{ij}(s) - \varepsilon\delta_{ij}(s)\right]ds\right\}\Bigg(c_i^\varepsilon(t) - m_i(t) \\ \left. + \sum_{j\neq i}\left[\lambda_{ij}(t) - \varepsilon\delta_{ij}(t)\right]\overline{W}^\varepsilon(t,j)\Bigg)dt\right|_{\varepsilon=0}$$

$$= \int_0^T e^{-rt} \left(\int_0^t \sum_{j \neq i} \delta_{ij}(s)\, ds \right) \tilde{S}(i,t) \left(c_i(t) - m_i(t) + \sum_{j \neq i} \lambda_{ij}(t) \overline{W}(t,j) \right) dt$$
$$- e^{-rt} \tilde{S}(i,t) \sum_{j \neq i} \delta_{ij}(t)\, \overline{W}(t,j)$$
$$+ e^{-rt} \tilde{S}(i,t) \left(\left. \frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon} \right|_{\varepsilon=0} + \sum_{j \neq i} \lambda_{ij}(t) \left. \frac{\partial \overline{W}^{\varepsilon}(t,j)}{\partial \varepsilon} \right|_{\varepsilon=0} \right) dt$$

Substituting in then yields the final result for the marginal utility of the reduction in this transition intensity:

$$\left. \frac{\partial \mathbb{E}U}{\partial \varepsilon} \right|_{\varepsilon=0} = \int_0^T e^{-\rho t} \left(\int_0^t \sum_{j \neq i} \delta_{ij}(s)\, ds \right) \tilde{S}(i,t) \left(u\big(c_i(t), q_i(t)\big) + \sum_{j \neq i} \lambda_{ij}(t)\, V\big(t, \overline{W}_t, j\big) \right)$$
$$- e^{-\rho t} \tilde{S}(i,t) \sum_{j \neq i} \delta_{ij}(t)\, V\big(t, \overline{W}_t, j\big)$$
$$- \theta e^{-rt} \left(\int_0^t \sum_{j \neq i} \delta_{ij}(s)\, ds \right) \tilde{S}(i,t) \left(c_i(t) - m_i(t) + \sum_{j \neq i} \lambda_{ij}(t) \overline{W}(t,j) \right)$$
$$+ \theta e^{-rt} \tilde{S}(i,t) \sum_{j \neq i} \delta_{ij}(t)\, \overline{W}(t,j) dt$$
$$= \int_0^T \left(e^{-\rho t} u(c_i(t), q_i(t)) + \sum_{j \neq i} \lambda_{ij}(t)\, V\big(t, \overline{W}_t, j\big) \right.$$
$$\left. + \theta e^{-rt} \left(m_i(t) - c_i(t) - \sum_{j \neq i} \lambda_{ij}(t)\, \overline{W}(t,j) \right) \right) \left(\int_0^t \sum_{j \neq i} \delta_{ij}(s)\, ds \right) \tilde{S}(i,t)$$
$$- \left(e^{-\rho t} \sum_{j \neq i} \delta_{ij}(t)\, V\big(t, \overline{W}_t, j\big) - \theta e^{-rt} \sum_{j \neq i} \delta_{ij}(t)\, \overline{W}(t,j) \right) \tilde{S}(i,t) dt$$

The first term inside the integral of the above expression represents the gain in marginal utility from a reduction in the probability of exiting state $Y_t = i$, and is analogous to the expression for $\left. \frac{\partial \mathbb{E}U}{\partial \varepsilon} \right|_{\varepsilon=0}$ for life-extension. The second term represents the loss in marginal utility from the reduction in probability of transitioning to other possible states. If these other states correspond to lower health (utility) than state i, then the net effect on marginal utility is positive.

Next, we choose the Delta-Dirac function for $\delta(\cdot)$, so that the probability is perturbed at $t = 0$ and remains unaffected otherwise. We also consider a reduction in the transition probability for only one alternative state, j_0, so that $\delta_{ij}(t) = 0\ \forall j \neq j_0$. This simplifies the above expression to

$$\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T \left(e^{-\rho t} u\big(c_i(t), q_i(t)\big) + \sum_{j\neq i} \lambda_{ij}(t)\, V\big(t, \overline{W}_t, j\big) + \theta e^{-rt} \left(m_i(t) - c_i(t) - \sum_{j\neq i} \lambda_{ij}(t)\, \overline{W}(t,j) \right) \right) \tilde{S}(i,t)$$

$$- \left(e^{-\rho t} V\big(t, \overline{W}_t, j_0\big) - \theta e^{-rt} \overline{W}(t, j_0) \right) \tilde{S}(i,t) dt$$

$$= \int_0^T \left(e^{-\rho t} \tilde{S}(i,t) u\big(c_i(t), q_i(t)\big) + \sum_{j\neq i} \lambda_{ij}(t)\, V\big(t, \overline{W}_t, j\big) \right) dt - V\big(0, \overline{W}_t, j_0\big)$$

$$+ \theta \left[\int_0^T \left(e^{-rt} \left(m_i(t) - c_i(t) - \sum_{j\neq i} \lambda_{ij}(t)\, \overline{W}(t,j) \right) \right) \tilde{S}(i,t) dt + \overline{W}(0, j_0) \right]$$

$$= \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) u\left(c_{Y_t}(t), q_{Y_t}(t) \right) dt \,\middle|\, Y_0 = i \right] - \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) u\big(c_{Y_t}(t), q_{Y_t}(t) \big) dt \,\middle|\, Y_0 = j_0, W_0^* = W^{new} \right]$$

$$+ \theta \left(\mathbb{E}\left[\int_0^T e^{-rt} S(t) \left(m_{Y_t}(t) - c_{Y_t}(t) \right) dt \,\middle|\, Y_0 = i \right] \right.$$

$$\left. - \mathbb{E}\left[\int_0^T e^{-rt} S(t) \left(m_{Y_t}(t) - c_{Y_t}(t) \right) dt \,\middle|\, Y_0 = j_0, W_0^* = W^{new} \right] \right)$$

where W^{new} represents the change in value of the annuity menu purchased in state i when immediately jumping to state j_0. Dividing the above expression by the marginal utility of wealth, given by (15), then yields (17), the value of statistical illness (VSI).

QED

The proof of **Proposition 6** makes use of the following lemma.

Appendix Lemma A1:

When consumers are not annuitized, the expected marginal utility of consumption in state i at time t is equal to:

$$u_c\big(c_i(t), q_i(t)\big) = \mathbb{E}\left[e^{(r-\rho)(\tau - t)} \exp\left\{ - \int_t^\tau \mu(s) ds \right\} u_c\big(c_{Y_\tau}(\tau), q_{Y_\tau}(\tau) \big) \,\middle|\, Y_t = i \right]$$

Proof of Appendix Lemma A1:

The proof proceeds by induction on $i \leq n$. For the base case $i = n$, in which no state transitions are possible, the solution to the adjoint equation (given in the main text) simplifies to:

$$\begin{aligned} p_\tau^{(n)} &= \theta^{(n)} e^{-r\tau} = \exp\left\{ - \int_0^\tau \rho + \overline{\mu}_n(s) ds \right\} u_c(c_n(\tau), q_n(\tau)) \\ &= \theta^{(n)} e^{-rt} e^{-r(\tau - t)} \\ &= p_t^{(n)} e^{-r(\tau - t)} \\ &= \exp\left\{ - \int_0^t \rho + \overline{\mu}_n(s) ds \right\} u_c(c_n(t), q_n(t)) e^{-r(\tau - t)} \end{aligned}$$

This then implies that

$$u_c(c_n(t), q_n(t)) = e^{r(\tau - t)} e^{-\rho(\tau - t)} \exp\left\{ - \int_t^\tau \overline{\mu}_n(s) ds \right\} u_c(c_n(\tau), q_n(\tau))$$

which shows that the lemma holds for $i = n$.

For the induction step, suppose the lemma is true for case i. For any subinterval $[0, \tau]$, the solution of the adjoint equation can be written as:

$$p_t^{(i)} = \left[\int_t^\tau e^{(r-\rho)s} \exp\left\{-\int_0^s \overline{\mu}_i(u) + \sum_{j\neq i} \lambda_{ij}(u)\, du\right\} \sum_{j\neq i} \lambda_{ij}(s) \frac{\partial V(s, W(s), j)}{\partial W(s)} ds\right] e^{-rt} + \theta(\tau, i) e^{-rt} \tag{A1}$$

where $\theta(\tau, i)$ is a constant that depends on the choice of τ and i. (Take the derivative of $p_t^{(i)}$ with respect to t to verify.) Evaluating equation (A1) at $t = \tau$ and combining with equation (20) from the main text yields:

$$p_\tau^{(i)} = \theta(\tau, i) e^{-r\tau} = \exp\left\{-\int_0^\tau \rho + \overline{\mu}_i(s) + \sum_{j\neq i} \lambda_{ij}(s)\, ds\right\} u_c\big(c_i(\tau), q_i(\tau)\big)$$

which implies

$$\theta(\tau, i) = e^{(r-\rho)\tau} \exp\left\{-\int_0^\tau \overline{\mu}_i(s) + \sum_{j\neq i} \lambda_{ij}(s)\, ds\right\} u_c\big(c_i(\tau), q_i(\tau)\big) \tag{A2}$$

Also, from equation (20) we know that:

$$p_t^{(i)} = \exp\left\{-\int_0^t \rho + \overline{\mu}_i(s) + \sum_{j\neq i} \lambda_{ij}(s)\, ds\right\} u_c\big(c_i(t), q_i(t)\big)$$

Plugging equations (20) and (A2) into equation (A1) yields:

$$\begin{aligned} u_c\big(c_i(t), q_i(t)\big) \exp\left\{-\int_0^t \rho + \overline{\mu}_i(s) + \sum_{j\neq i} \lambda_{ij}(s)\, ds\right\} \\ = \left[\int_t^\tau e^{(r-\rho)s} \exp\left\{-\int_0^s \overline{\mu}_i(u) + \sum_{j\neq i} \lambda_{ij}(u)\, du\right\} \sum_{j\neq i} \lambda_{ij}(s) \frac{\partial V(s, W(s), j)}{\partial W(s)} ds\right] e^{-rt} \\ + e^{-rt} e^{(r-\rho)\tau} \exp\left\{-\int_0^\tau \overline{\mu}_i(s) + \sum_{j\neq i} \lambda_{ij}(s)\, ds\right\} u_c\big(c_i(\tau), q_i(\tau)\big) \end{aligned}$$

Since $\frac{\partial V(s, W(s), j)}{\partial W(s)} = u_c\big(c_j(s), q_j(s)\big)$, we obtain:

$$\begin{aligned} u_c\big(c_i(t), q_i(t)\big) = \int_t^\tau e^{(r-\rho)(s-t)} \exp\left\{-\int_t^s \overline{\mu}_i(u) + \sum_{j\neq i} \lambda_{ij}(u)\, du\right\} \sum_{j\neq i} \lambda_{ij}(s) u_c\big(c_j(s), q_j(s)\big)\, ds \\ + e^{(r-\rho)(\tau-t)} \exp\left\{-\int_t^\tau \overline{\mu}_i(s) + \sum_{j\neq i} \lambda_{ij}(s)\, ds\right\} u_c\big(c_i(\tau), q_i(\tau)\big) \end{aligned}$$

$$= \int_t^\tau e^{(r-\rho)(s-t)} \exp\left\{-\int_t^s \overline{\mu}_i(u) + \sum_{j\neq i} \lambda_{ij}(u)\, du\right\} \sum_{j\neq i} \lambda_{ij}(s) \mathbb{E}\left[e^{(r-\rho)(\tau-s)} \exp\left\{-\int_s^\tau \mu(s) ds\right\} u_c\big(c_{y_t}(\tau), q_{y_t}(\tau)\big)\middle| Y_s = j\right] ds$$
$$+ e^{(r-\rho)(\tau-t)} \exp\left\{-\int_t^\tau \overline{\mu}_i(s) + \sum_{j\neq i} \lambda_{ij}(s)\, ds\right\} u_c\big(c_i(\tau), q_i(\tau)\big)$$
$$= \mathbb{E}\left[e^{(r-\rho)(\tau-s)} \exp\left\{-\int_t^\tau \mu(s) ds\right\} u_c\big(c_{y_t}(\tau), q_{y_t}(\tau)\big)\middle| Y_t = i\right]$$

where the second equality follows from the induction hypothesis.

QED

Proof of Proposition 6:

Choosing once again the Delta-Dirac function for $\delta(\cdot)$ in **Lemma 5** yields

$$\left.\frac{\partial \mathbb{E}U}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T \left[e^{-\rho t} \tilde{S}(i,t) \left(u\big(c_i(t), q_i(t)\big) + \sum_{j\neq i} \lambda_{ij}(t)\, V(t, W(t), j) \right)\right] dt$$
$$= \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) u\big(c_{y_t}(t), q_{y_t}(t)\big) dt \middle| Y_0 = i\right]$$

Dividing the result by the marginal utility of wealth at time $t = 0$ then yields the value of statistical life given by equation (21):

$$VSL = \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) \frac{u\big(c_{y_t}(t), q_{y_t}(t)\big)}{u\big(c_{Y_0}(0), q_{Y_0}(0)\big)} dt \middle| Y_0 = i\right] = \int_0^T e^{-rt} v_i(t) dt$$

Applying **Appendix Lemma A1** for $t = 0$ allows us to rewrite VSL as

$$VSL_i = \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) \frac{u\big(c_{y_t}(t), q_{y_t}(t)\big)}{\mathbb{E}\left[e^{(r-\rho)t} \exp\left\{-\int_0^t \mu(s) ds\right\} u_c\big(c_{Y_\tau}(\tau), q_{Y_\tau}(\tau)\big)\middle| Y_0\right]} dt \middle| Y_0 = i\right]$$
$$= \mathbb{E}\left[\int_0^T e^{-rt} \frac{S(t) u\big(c_{y_t}(t), q_{y_t}(t)\big)}{\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s) ds\right\} u_c\big(c_{Y_\tau}(\tau), q_{Y_\tau}(\tau)\big)\middle| Y_0\right]} dt \middle| Y_0 = i\right]$$

which by exchanging expectation and integration shows that the value of a life-year, $v_i(t)$, is equal to the expected utility of consumption normalized by the expected marginal utility of consumption:

$$v_i(t) = \frac{\mathbb{E}\left[S(t) u\left(c_{y_t}(t), q_{y_t}(t)\right)\middle| Y_0 = i\right]}{\mathbb{E}\left[S(t) u_c\left(c_{y_t}(t), q_{y_t}(t)\right)\middle| Y_0 = i\right]}$$

QED

Proof of Proposition 7:

The proposition assumes there are $n = 2$ states, with $\overline{\mu}_2(s) > \overline{\mu}_1(s) \forall s$. That is, health in state 2 is strictly worse than health in state 1. For simplicity, we abstract from quality of life, $q(t)$. Without loss of generality, we will prove the proposition for the case where the consumer transitions from state 1 to state 2 at time $t = 0$.

For state 2, the solution to the adjoint equation is:

$$p_t^{(2)} = \theta^{(2)} e^{-rt}$$

and from the first-order condition (20) we obtain:

$$p_t^{(2)} = e^{-\rho t} \exp\left\{\int_0^t \overline{\mu}_2(t) ds\right\} u_c\big(c_2(t)\big)$$

The two preceding equations imply that

$$u_c\big(c_2(t)\big) = \theta^{(2)} e^{(\rho - r)t} \exp\left\{-\int_0^t \overline{\mu}_2(t) ds\right\}$$

For state 1, the adjoint equation is:

$$\begin{aligned}
\dot{p}_t^{(1)} &= -p_t^{(1)} r - e^{-\rho t} \exp\left\{\int_0^t \overline{\mu}_1(t) + \lambda_{12}(s) ds\right\} \lambda_{12}(t) \frac{\partial V(t, W(t), 2)}{\partial W(t)} \\
&= -p_t^{(1)} r - e^{-\rho t} \exp\left\{\int_0^t \overline{\mu}_1(t) + \lambda_{12}(s) ds\right\} \lambda_{12}(t) u_c\big(c_2(t)\big) \\
&= -p_t^{(1)} r - e^{-rt} \exp\left\{-\int_0^t \lambda_{12}(s) ds\right\} \lambda_{12}(t) \theta^{(2)} \exp\left\{\int_0^t \overline{\mu}_2(t) - \overline{\mu}_1(t) ds\right\}
\end{aligned} \tag{A3}$$

Before proceeding, we first prove the following two lemmas.

Appendix Lemma A2:

There exists a $t \in [0, T]$ such that

$$p_t^{(1)} \geq \exp\left\{\int_0^t \lambda_{12}(s) ds\right\} p_t^{(2)}$$

Proof of Appendix Lemma A2:

Suppose by way of contradiction that $p_t^{(1)} < \exp\left\{\int_0^t \lambda_{12}(s) ds\right\} p_t^{(2)}$ $\forall t \in [0, T]$. Then, since $\overline{\mu}_2(s) > \overline{\mu}_1(s)$ we have

$$e^{-\rho t} \exp\left\{\int_0^t \overline{\mu}_2(t) ds\right\} p_t^{(1)} < e^{-\rho t} \exp\left\{\int_0^t \overline{\mu}_1(t) ds\right\} \exp\left\{\int_0^t \lambda_{12}(s) ds\right\} p_t^{(2)}$$

Rearranging then yields

$$u_c(c_1(t)\,) = \frac{p_t^{(1)}}{e^{-\rho t} \exp\left\{\int_0^t \overline{\mu}_1(t) ds\right\} \exp\left\{\int_0^t \lambda_{12}(s) ds\right\}} < \frac{p_t^{(2)}}{e^{-\rho t} \exp\left\{\int_0^t \overline{\mu}_2(t) ds\right\} p_t^{(1)}} = u_c(c_2(t)\,)$$

which implies $c_2(t) < c_1(t) \forall t$. But then we have a contradiction: $c_2(t)$ cannot be an optimal consumption plan because the feasible consumption plan $c_1(t)$ strictly dominates $c_2(t)$.

QED

Appendix Lemma A3:

$$p_0^{(1)} > \theta^{(2)} = p_0^{(2)}$$

Proof of Appendix Lemma A3:

Define

$$g(t) = \exp\left\{-\int_0^t r + \lambda_{12}(s)ds\right\}\theta^{(2)} = \exp\left\{-\int_0^t \lambda_{12}(s)ds\right\}p_t^{(2)}$$

Differentiating with respect to t yields

$$\begin{aligned}\dot{g}(t) &= -g(t)r - \exp\left\{-rt - \int_0^t \lambda_{12}(s)ds\right\}\lambda_{12}(t)\theta \\ &= \phi(g(t), t)\end{aligned}$$

*Combining this result with equation (*A3*) then yields the following inequality:*

$$\dot{p}_t^{(1)} < \phi\left(p_t^{(1)}, t\right)$$

Suppose by way of contradiction that $p_0^{(1)} < \theta = g(0)$. *Then by standard comparison arguments for ordinary differential equations, we have* $p_t^{(1)} < g(t) = \exp\left\{-\int_0^t \lambda_{12}(s)ds\right\}p_t^{(2)}$ *, which is a contradiction to the result from* ***Appendix Lemma A2****.*

QED

Thus, we have

$$u_c\big(c_1(0)\big) = p_0^{(1)} > p_0^{(2)} = u_c(c_2(0))$$

which implies

$$c_2(0) > c_1(0)$$

QED

B. Numerical models

B1. Deterministic mortality

We employ dynamic programming techniques to solve for the optimal consumption path. The value function is defined as:

$$V(t, w_t) = \max_{\{c_t\}} \sum_{s=t}^{T} e^{-\rho(s-t)} S_t(s) u(c_s)$$

We can use the value function to rewrite the optimization problem as a recursive Bellman equation:

$$V(t, w_t) = \max_{\{c_t\}} u(c_t) + \frac{1 - q_t}{e^\rho} V(t + 1, w_{t+1})$$

Once we have solved for the optimal consumption path, we can use the analytical formulas derived in the main text to calculate the value of life.

Our calibration exercises use numerical methods to solve the following Bellman equation:

$$V(t, w_t) = \max_{\{c_t\}} u(c_t) + \frac{1 - q_t}{e^\rho} V(t + 1, w_{t+1})$$

Because the problem is finite, we can work backwards from the final period. We discretize the state space into $N_w = 2{,}000$ points evenly distributed across the interval $[0, w_{max}]$. Let that set of values be $\{w_n\}$. Define $g_t(w_t) = w_{t+1}$ as a mapping from the current wealth state, w_t, to the optimal wealth state in the following period, w_{t+1}.

It is clear that the consumer should consume all her wealth in the final period, i.e., $g_T(w_T) = 0$ for all $w_T \in \{w_n\}$. This implies that $V(T, w_T) = u(w_T + m_T)$ for all $w_T \in \{w_n\}$.

Next, we calculate $V(T-1, w_{T-1}) = \max_{g(w_{T-1}) = w_T} u(w_{T-1} + m_{T-1} - w_T/(1+r)) + \frac{1-q_{t+1}}{e^\rho} V(T, w_T)$. In other words, for each $w_{T-1} \in \{w_n\}$, we calculate the optimal $V(T-1, w_{T-1})$ by determining which choice of $g_{T-1}(w_{T-1}) = w_T \in \{w_n\}$ will maximize utility. This algorithm is then repeated for $t = T - 2, T - 3, \ldots 1$.

Given the initial condition, w_1, we can then employ our results to calculate $w_2 = g_1(w_1)$, $w_3 = g_2(w_2)$,… w_T. Period consumption, c_t, is then calculated using the equation for the budget constraint.

B2. Stochastic mortality

We focus on the case where the consumer does not have access to annuities. We ignore income, and assume that all of consumer's wealth is available at time $t = 0$. This will allow us to generate an analytic solution to the consumer's problem, given by:

$$\max_{\{c_t\}} \mathbb{E}_0 \left[\sum_{t=0}^{T} e^{-\rho t} S_0(t) u(c_t) \right]$$

where

$$w_0 \ given$$

$$w_t = (w_{t-1} - c_{t-1}) e^r, w_t \geq 0$$

The utility function is

$$u(c) = \frac{c^{1-\gamma} - \underline{c}^{1-\gamma}}{1 - \gamma}$$

Because optimal consumption is unaffected by affine transformations of utility, we will assume $u(c) = c^{1-\gamma}/(1-\gamma)$ when solving the model for consumption.

Define the value function

$$V(t, w_t, Y_t) = \max_{\{c_s\}} \mathbb{E} \left[\sum_{s=t}^{T} e^{-\rho(s-t)} S_t(s) u(c_s) \middle| Y_t, w_t, alive \right]$$

subject to

$$w_s = (w_{s-1} - c_{s-1}) e^r, s > t, w_s \geq 0$$

Then we obtain the following Bellman equation:

$$V(t,w,i)=\max_{c_t}\left\{u(c_t)+e^{-\rho}\left(1-\overline{q}_i(t)\right)\sum_{j=1}^{n}p_{ij}(t)V(t+1,(w-c_t)e^r,j)\right\}$$

where $V(T,w,i)=u(w)=w^{1-\gamma}/(1-\gamma)$.

Appendix Proposition B1:

The value function and the optimal consumption level satisfy

$$V(t,w,i)=\frac{w^{1-\gamma}}{1-\gamma}K_{t,i}$$

$$c^*(t,w,i)=w\cdot c_{t,i}$$

where for $t<T$:

$$c_{t,i}=\frac{e^r+\dfrac{(\rho-r)}{\gamma}\left(1-\overline{q}_i(t)\right)^{-1/\gamma}\left(\sum_{j=1}^{n}p_{ij}(t)K_{t+1,j}\right)^{-1/\gamma}}{1+e^{r+\frac{\rho-r}{2}}\left(1-\overline{q}_i(t)\right)^{-1/\gamma}\left(\sum_{j=1}^{n}p_{ij}(t)\,K_{t+1,j}\right)^{-1/\gamma}}$$

$$c_{T,i}=1$$

and $K_{t,i}$ satisfies the recursion:

$$K_{t,i}=\frac{\left[e^{-r\gamma-(\rho-r)}\left(1-\overline{q}_i(t)\right)\sum_{j=1}^{n}p_{ij}(t)K_{t+1,j}\right]^{1-1/\gamma}+e^{-r\gamma-(\rho-r)}\left(1-\overline{q}_i(t)\right)\sum_{j=1}^{n}p_{ij}(t)K_{t+1,j}}{\left[1+\left[e^{-r\gamma-(\rho-r)}\left(1-\overline{q}_i(t)\right)\sum_{j=1}^{n}p_{ij}(t)K_{t+1,j}\right]^{-1/\gamma}\right]^{1-\gamma}}$$

$$K_{T,i}=1$$

Proof of Appendix Proposition B1: see end of appendix B

When calculating VSL, we incorporate subsistence consumption back into the utility function. From the theory presented in the main text of the paper, we obtain:

$$\begin{aligned}VSL&=\mathbb{E}\left[\sum_{t=0}^{T}\exp\left\{-\int_0^t\rho+\mu(s)ds\right\}\frac{u\left(c_{Y_t}(t)\right)}{u_c\left(c_{Y_0}(0)\right)}\middle|Y_0\right]\\&=\sum_{t=0}^{T}e^{-rt}\frac{\mathbb{E}\left[\exp\left\{-\int_0^t\mu(s)ds\right\}u\left(c_{Y_t}(t)\right)\middle|Y_0\right]}{\mathbb{E}\left[\exp\left\{-\int_0^t\mu(s)ds\right\}u_c\left(c_{Y_t}(t)\right)\middle|Y_0\right]}\\&=\sum_{t=0}^{T}e^{-rt}\frac{\mathbb{E}\left[\exp\left\{-\int_0^t\mu(s)ds\right\}\left(\dfrac{c_{Y_t}^{1-\gamma}}{1-\gamma}-\dfrac{\underline{c}^{1-\gamma}}{1-\gamma}\right)\middle|Y_0\right]}{\mathbb{E}\left[\exp\left\{-\int_0^t\mu(s)ds\right\}c_{Y_t}^{-\gamma}\middle|Y_0\right]}\end{aligned}$$

or

$$VSL=\frac{1}{1-\gamma}\sum_{t=0}^{T}e^{-rt}\underbrace{\frac{\mathbb{E}\left[\exp\left\{-\int_0^t\mu(s)ds\right\}c_{Y_t}^{1-\gamma}\middle|Y_0\right]-\underline{c}^{1-\gamma}\mathbb{E}\left[\exp\left\{-\int_0^t\mu(s)ds\right\}\middle|Y_0\right]}{\mathbb{E}\left[\exp\left\{-\int_0^t\mu(s)ds\right\}c_{Y_t}^{-\gamma}\middle|Y_0\right]}}_{\overline{v}(t)}$$

To evaluate this expression for VSL, we will make use of the following lemma.

Appendix Lemma B2: Let $W_{t,j}(\Psi) = \mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\} w_t{}^{\Psi}\mathbf{1}\{Y_t = j\}\middle| Y_0\right]$ for $Y \in (1,\infty)$. Then $W_{t,j}(\Psi)$ satisfies the following recursion:

$$W_{0,Y_0}(\Psi) = w_0^{\Psi}, W_{0,i}(\Psi) = 0, i \neq Y_0$$
$$W_{t+1,j}(\Psi) = e^{r\Psi}\sum_{k=1}^{n} W_{t,k}(\Psi)\left(1 - c_{t,k}\right)^{\Psi}\left(1 - \overline{q}_k(t)\right)p_{k,j}(t)$$

Proof of Appendix Lemma B2: see end of appendix B

Note that for $\Psi = 0$, the expression $\sum_{j=1}^{n} W_{t,j}(0) = \mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\}\middle| Y_0\right]$ is simply the t-year survival probability. Using this **Appendix Lemma B2**, we obtain:

Appendix Proposition B3:

$$VSL_{Y_0} = \frac{1}{1-\gamma}\sum_{t=0}^{T} e^{-rt}\underbrace{\frac{\sum_{j=1}^{n} c_{t,j}^{1-\gamma} W_{t,j}(1-\gamma) - \underline{c}^{1-\gamma}\sum_{j=1}^{n} W_{t,j}(0)}{\sum_{j=1}^{n} c_{t,j}^{-\gamma} W_{t,j}(-\gamma)}}_{\overline{v}(t)}$$

Proof of Appendix Proposition B3: see end of appendix B

We also immediately obtain the following corollary:

Appendix Corollary B4:

$$\begin{aligned} VSI_{i,j} &= VSL_i - VSL_j \frac{c_{j,0}^{-\gamma}}{c_{i,0}^{-\gamma}} \\ &= VSL_i - \left(\frac{c_{i,0}}{c_{j,0}}\right)^{\gamma} VSL_j \end{aligned}$$

Proofs for Appendix B2

Proof of Appendix Proposition B1:

The proof proceeds by induction on $t \leq T$. For the base case $t = T$, we set $K_{T,i} = 1$. For the induction step, suppose the proposition is true for case $t + 1$. We have

$$V(t, w, i) = \max_c \left\{ \frac{c^{1-\gamma}}{1-\gamma} + e^{-\rho} \left(1 - \overline{q}_i(t)\right) \sum_{j=1}^{n} p_{ij}(t) \frac{K_{t+1,j}}{1-\gamma} [(w - c)e^r]^{1-\gamma} \right\}$$

From the first-order condition we obtain:

$$c^{-\gamma} = e^{r-\rho} \left(1 - \overline{q}_i(t)\right) \sum_{j=1}^{n} p_{ij}(t) K_{t+1,j} [(w - c)e^r]^{-\gamma}$$

Rearranging yields

$$\begin{aligned} c &= (w - c)e^r e^{(\rho-r)/\gamma} \left(1 - \overline{q}_i(t)\right)^{-1/\gamma} \left(\sum_{j=1}^{n} p_{ij}(t) K_{t+1,j}\right)^{-1/\gamma} \\ &= w e^{r+(\rho-r)/\gamma} \left(1 - \overline{q}_i(t)\right)^{-1/\gamma} \left(\sum_{j=1}^{n} p_{ij}(t) K_{t+1,j}\right)^{-1/\gamma} \\ &\qquad - c e^{r+(\rho-r)/\gamma} \left(1 - \overline{q}_i(t)\right)^{-1/\gamma} \left(\sum_{j=1}^{n} p_{ij}(t) K_{t+1,j}\right)^{-1/\gamma} \\ &= w \underbrace{\frac{e^{r+(\rho-r)/\gamma} \left(1 - \overline{q}_i(t)\right)^{-1/\gamma} \left(\sum_{j=1}^{n} p_{ij}(t) K_{t+1,j}\right)^{-1/\gamma}}{1 + e^{r+(\rho-r)/\gamma} \left(1 - \overline{q}_i(t)\right)^{-1/\gamma} \left(\sum_{j=1}^{n} p_{ij}(t) K_{t+1,j}\right)^{-1/\gamma}}}_{c_{t,i}} \end{aligned}$$

Thus we obtain:

$$\begin{aligned} V(t, w, i) &= \frac{w^{1-\gamma}}{1-\gamma} c_{t,i}^{1-\gamma} + e^{-\rho} \left(1 - \overline{q}_i(t)\right) \sum_{j=1}^{n} p_{ij}(t) \frac{K_{t+1,j}}{1-\gamma} \left[\underbrace{we^r - e^r w c_{t,i}}_{we^r(1-c_{t,i})}\right]^{1-\gamma} \\ &= \frac{w^{1-\gamma}}{1-\gamma} \left[c_{t,i}^{1-\gamma} + e^{(1-\gamma)r-\rho} \left(1 - c_{t,i}\right)^{1-\gamma} \left(1 - \overline{q}_i(t)\right) \sum_{j=1}^{n} p_{ij}(t) K_{t+1,j}\right] \\ &= \frac{w^{1-\gamma}}{1-\gamma} \left[\frac{\left[e^{-r\gamma-(\rho-r)} \left(1 - \overline{q}_i(t)\right) \sum_{j=1}^{n} p_{ij}(t) K_{t+1,j}\right]^{1-1/\gamma} + e^{-r\gamma-(\rho-r)} \left(1 - \overline{q}_i(t)\right) \sum_{j=1}^{n} p_{ij}(t) K_{t+1,j}}{\left[1 + \left[e^{-r\gamma-(\rho-r)} \left(1 - \overline{q}_i(t)\right) \sum_{j=1}^{n} p_{ij}(t) K_{t+1,j}\right]^{-\frac{1}{\gamma}}\right]^{1-\gamma}}\right] \end{aligned}$$

QED

Proof of Appendix Lemma B2:

$$
\begin{aligned}
W_{t+1,j}(\Psi) = & \ \mathbb{E}\left[\exp\left\{-\int_0^{t+1}\mu(s)ds\right\}(w_{t+1})^{\Psi}\mathbf{1}\{Y_{t+1}=j\}\right] \\
= & \ \mathbb{E}\left[\exp\left\{-\int_0^{t}\mu(s)ds\right\}\left((w_t-c_t)e^r\right)^{\Psi}\mathbf{1}\{Y_{t+1}=j\}\exp\left\{-\int_t^{t+1}\mu(s)ds\right\}\right] \\
= & \sum_{k=1}^{n}\mathbb{E}\left[\mathbf{1}\{Y_t=k\}\exp\left\{-\int_0^{t}\mu(s)ds\right\}e^{r\Psi}w_t^{\Psi}\left(1-c_{t,k}\right)^{\Psi}\underbrace{\mathbb{E}\left[\mathbf{1}\{Y_{t+1}=j\}\exp\left\{-\int_t^{t+1}\mu(s)ds\right\}\middle| Y_t=k\right]}_{\left(1-\overline{q}_k(t)\right)p_{kj}(t)}\right] \\
= & \ e^{r\Psi}\sum_{k=1}^{n}W_{t,k}(\Upsilon)\left(1-c_{t,k}\right)^{\Psi}\left(1-\overline{q}_k(t)\right)p_{kj}(t)
\end{aligned}
$$

QED

Proof of Appendix Proposition B3:

Note that we have

$$
\begin{aligned}
\mathbb{E}\left[\exp\left\{-\int_0^{t}\mu(s)ds\right\}c_{Y_t}^{\Psi}\right] &= \sum_{j=1}^{n}\mathbb{E}\left[\exp\left\{-\int_0^{t}\mu(s)ds\right\}c_{Y_t}^{\Psi}\mathbf{1}\{Y_t=j\}\right] \\
&= \sum_{j=1}^{n}\mathbb{E}\left[\exp\left\{-\int_0^{t}\mu(s)ds\right\}c_{t,j}^{\Psi}w_t^{\Psi}\mathbf{1}\{Y_t=j\}\right] \\
&= \sum_{j=1}^{n}c_{t,j}^{\Psi}\underbrace{\mathbb{E}\left[\exp\left\{-\int_0^{t}\mu(s)ds\right\}w_t^{\Psi}\mathbf{1}\{Y_t=j\}\right]}_{w_{t,j}(\Psi)}
\end{aligned}
$$

The proof follows by setting $\Psi=1-\gamma$, 0, and $-\gamma$ in the expression for VSL.

QED