Mortality Risk, Insurance, and the Value of Life*

Daniel Bauer
University of Wisconsin-Madison

Darius Lakdawalla
University of Southern California and NBER

Julian Reif
University of Illinois and NBER

Abstract. We develop and apply a generalized framework for valuing health and longevity improvements that departs from conventional assumptions of full annuitization and deterministic mortality. In contrast to conventional theory, we find a given mortality improvement may be worth *more*, not less, to patients facing shorter lives. Using real-world data, we calculate that severe illness can increase the value of statistical life by over $1 million. This result reconciles an anomaly in the research on preferences for life-extension. Moreover, our framework can value the prevention of mortality *and* of illness. We calculate that treating illness is up to an order of magnitude more valuable to consumers than prevention, even when both extend life equally. This asymmetry helps explain low observed investment in preventive care. Finally, we show that retirement annuities boost aggregate demand for life-extension. For instance, Social Security adds $11.5 trillion (10.5 percent) to the value of post-1940 longevity gains.

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I. INTRODUCTION

The economic analysis of risks to life and health has made enormous contributions to both academic discussions and public policy. Economists have used the standard tools of life-cycle consumption theory to propose a transparent framework that measures the value of improvements to both health and longevity. Economic concepts such as the value of statistical life play central roles in public policy discussions surrounding investments in medical care, public safety, environmental hazards, and countless other arenas.

The standard framework, however, assumes full annuitization and deterministic mortality risk. While analytically convenient and useful for illustrating some of the underlying economics, these assumptions are not realistic: it is well known that most people are far from fully annuitized (Brown et al. 2008), and that mortality risk depends on one's health state. Moreover, these assumptions hamper explanatory power in several ways: the standard framework cannot investigate what happens to the value of life upon falling ill, cannot meaningfully distinguish between preventive care and medical treatment, and glosses over policy-relevant relationships between the value of life and the structure of the annuity market. These issues are empirically relevant. Prior research suggests the value placed on life-extension varies considerably with health state (Nord et al. 1995). And, an array of evidence suggests that society invests less in preventive care than in medical treatment, even when both have the same consequences for health and longevity (Weisbrod 1991; Dranove 1998; Pryor and Volpp 2018).

This paper develops a general economic framework for valuing health improvements and applies it to data. We establish three main results. First, we derive conditions under which the value of life can *rise* following a negative health shock, and we demonstrate that this effect is economically significant. For example, we calculate that the value of statistical life (VSL) for a 70-year-old soars by over $1 million (50 percent) following the development of chronic conditions that impair her everyday living. Second, we introduce the value of statistical illness (VSI), which captures the willingness to pay to avoid sickness and includes VSL as a special case. We calculate that—holding wealth constant—a sick individual's initial willingness to pay for medical treatment is several times greater than a healthy individual's willingness to pay for preventive care that improves longevity by the same amount. Third, we calculate that the US Social Security program adds $11.5 trillion (10.5 percent) to the value of post-1940 longevity gains.

Incomplete annuitization drives all three of these results. A very simple example illustrates the intuition. Imagine a 60-year-old retiree with no bequest motive and a flat optimal consumption profile. If she fully annuitizes her savings, her consumption remains flat at, say, $30,000 annually. Now suppose she cannot annuitize any of her wealth. In this case, it is well known that the optimal consumption profile shifts forward (Yaari 1965), in response to the risk of dying with money still left in the bank (see Figure 1). Because VSL depends greatly on consumption, it too will shift forward. Thus, reductions in annuitization lower VSL at older ages, and increase VSL at younger ages. Conversely, retirement savings programs such as Social Security that increase annuitization levels will raise VSL at older ages and lower it at younger ages.

Our other results follow from the simple observation that it is optimal for an incompletely annuitized individual to shift her consumption forward, i.e., to spend down her wealth, following an adverse shock to life expectancy. At least for some initial period of time, the shock increases consumption, and thus reduces the marginal utility of consumption. An important insight of our paper is that although a negative shock to longevity always reduces lifetime utility, the accompanying reduction in the contemporaneous marginal utility of consumption can be large enough to cause VSL to *increase* even though life expectancy has fallen. Indeed, we show using real-world data that VSL is frequently higher for an individual diagnosed with a more fatal illness. Similarly, a sick individual's willingness to pay for treatment is frequently higher using real-world data than a healthy individual's willingness to pay for preventive care, even when both add the same number of life-years. This is in stark contrast to the conventional model with full annuitization, where a reduction in longevity always reduces VSL.

The first half of this paper provides a formal framework that yields these insights. We first demonstrate that consumption increases following an adverse shock to longevity, and we then derive sufficient conditions under which that shock also generates an accompanying increase in VSL.[1] These conditions are satisfied by the standard CRRA preferences used in prior value of life studies. We focus on mortality shocks, but our framework allows for shocks to quality of life and income as well. We then show how our framework leads to a more general concept, the value of statistical illness, which can be interpreted as an individual's willingness to pay for a marginal decrease in the risk of acquiring an illness. This allows us to compare the value of prevention to the value of treatment. In general, prevention and treatment are not valued equally unless consumers are fully annuitized. If VSL rises following a health shock, then the value of treatment can exceed the value of prevention. This result sheds new light on why consumers, firms, and health insurers appear reluctant to invest in prevention, even when there are considerable private life expectancy benefits (Weisbrod 1991; Dranove 1998; Pryor and Volpp 2018).

The second half of the paper applies our model to data. Our first empirical exercise incorporates detailed microsimulation data from the Future Elderly Model into a stochastic life-cycle model that allows mortality and quality of life to vary across 20 different health states. We demonstrate that our key theoretical result—

[1] The sign depends on whether the loss in lifetime utility is offset by a corresponding decrease in marginal utility. Specifically, an adverse mortality shock increases VSL when demand for current consumption is sufficiently inelastic, or when the marginal utility of demand is sufficiently linear (as measured by relative prudence).

that VSL can rise when life expectancy falls—is economically significant under reasonable parameterizations. For instance, we calculate that VSL rises from $2.9 million to $4.3 million for a 70-year-old who suffers a debilitating health shock that reduces her life expectancy by nearly 7 years and also worsens her quality of life. This relationship between health shocks and VSL generates substantial variability in the aggregate: Monte Carlo simulations performed on a population of initially healthy 50-year-olds predict that health shocks generate an inter-vigintile (middle 90 percent) VSL range of $4.2 to $5.3 million by age 60. In addition, we show that longevity gains are more valuable in states with lower remaining life expectancy. Finally, we calculate that the value of treating life-threatening conditions like cancer is worth up to 10 times more than equivalent preventive care that adds the same number of years to an individual's life expectancy. Our results are robust to including wealth shocks and a bequest motive.

Our second exercise illustrates the connections between public annuity programs and the societal value of increases in longevity, defined as individuals' private willingness to pay for life-extension plus the effect of life-extension on expected future consumption and income. We calculate that Social Security adds $11.5 trillion (10.5 percent) to the value of post-1940 longevity gains, relative to a setting with no annuity markets, by raising the value of life at older ages. This gain is worth over $35,000 per person to the current population, or about half as much as the longevity insurance value of Social Security. Moreover, Social Security increases the aggregate value of potential future increases in longevity by over 10 percent, so that a 1 percent reduction in population-wide mortality is $138 billion more valuable than it would have been without the program. Increasing the size of Social Security pensions by 50 percent would add a further $72 billion of value to this mortality decline. Finally, we show that a strong bequest motive reduces the effect of Social Security on the value of longevity improvements by half. This suggests the effect of annuitization on the value of life matters most for low-income individuals, who are less likely to have a significant bequest motive.

The economic literature on the value of life reaches back to Schelling (1968) and includes seminal studies by Arthur (1981), Rosen (1988), Murphy and Topel (2006), and Hall and Jones (2007). A few studies have considered departures from the assumption of full annuitization, but only under specialized preferences (Shepard and Zeckhauser 1984; Ehrlich 2000; Ehrlich and Yin 2005). Our framework builds on this literature by providing expressions for VSL under general preferences. We also further extend the conventional model to accommodate stochastic health shocks by exploiting recent advances in the systems and control literature (Parpas and Webster 2013). We view our application of these tools as a useful demonstration for other researchers working in stochastic settings. Our more general setting leads to the novel finding that VSL can rise following a health shock, and it allows us to introduce the concept of VSI. To the best of our knowledge, we provide the first life-cycle analysis of the value of preventing illness.

Our findings have two significant implications for cost-effectiveness analysis, which governs the allocation of healthcare resources in many “single-payer” countries such as the United Kingdom and Canada (Dranitsaris and Papadopoulos 2015) and continues to grow in importance in the multi-payer US healthcare marketplace (Goldman, Nussbaum, and Linthicum 2016). First, conventional cost-effectiveness analysis assumes that the value of extending life is insensitive to the severity of illness: providing X aggregate (quality-adjusted) life-years by extending life slightly for a large population of hypertension patients is worth the same as providing X aggregate life-years by extending life substantially for a proportionally smaller population of cancer patients. Unless individuals are fully annuitized, this equivalence is incorrect in our framework. In fact, it is often more valuable to provide larger life expectancy gains to smaller populations. This suggests that the traditional cost-effectiveness approach underinvests in the treatment of the most life-threatening illnesses relative to less severe conditions. This insight is also consistent with survey data on how consumers view the value of life-extension (Nord et al. 1995; Green and Gerard 2009; Linley and Hughes 2013), and can better inform the way economists and healthcare payers assess the value of medical technologies.

Second, cost-effectiveness analysis traditionally values life-years gained by prevention and treatment equally (Drummond et al. 2015). However, in our model these values depend on baseline health status, which creates a wedge between prevention and treatment. In contrast to Benjamin Franklin’s adage that “an ounce of prevention is worth a pound of cure” (Labaree 1960), we find that treatment is frequently much more valuable to consumers than prevention, even when it produces the same longevity gain. Of course, this does not preclude the possibility of positive externalities, such as the “herd immunity” of vaccines, or the relative clinical or cost-effectiveness of prevention versus treatment. Rather, it implies that longevity gains of fixed size are more valuable when gained through treatment instead of prevention.[2]

The remainder of this paper is organized as follows. Section II reviews the predictions of the conventional theory on the value of life and demonstrates how relaxing its assumption of full annuitization alters these predictions. Section III then generalizes the framework further by allowing health and income to be stochastic. Section IV presents empirical analysis that: (1) shows how health shocks can increase the value of statistical life; (2) illustrates how more severe health shocks cause consumers to place higher value on a given mortality reduction; (3) calculates the value of preventing different kinds of illness; and (4) quantifies the effect of Social Security on the value of statistical life. Section V concludes.

[2] This valuation differential depends on the individual’s current health state and is therefore most relevant for assessing the value of current medical R&D. The difference in the values of preventives and treatments developed in the distant future is negligible because they are necessarily valued from an ex ante (healthy) perspective.

II. DETERMINISTIC MODEL

Consider an individual who faces mortality risk. We are interested in analyzing the value of a marginal reduction in this risk. We first quantify this value in the conventional setting where markets are complete and the consumer has access to actuarially fair annuities (Rosen 1988; Murphy and Topel 2006). We then repeat this exercise in a "Robinson Crusoe" economy where the consumer cannot purchase annuities to insure against her uncertain lifetime (Shepard and Zeckhauser 1984; Ehrlich 2000; Johansson 2002). We compare our findings for these two polar cases to illustrate the basic insights of the paper. We focus on improvements in longevity and their relationship to annuity insurance markets, but we allow for improvements in quality of life as well. Section III then extends the model to accommodate stochastic health shocks and introduces the value of statistical illness.

Although it is optimal for a consumer to fully annuitize in the canonical life-cycle model (Yaari 1965), real-world annuitization rates are quite low. This "annuity puzzle" is the subject of numerous papers. Many explanations have been suggested, but there is no consensus on what drives incomplete annuitization (Brown et al. 2008). Our study takes the low rate of annuitization as a given empirical fact and illustrates its significance for the value of life. Section IV uses a numerical model to probe the sensitivity of our results to different assumptions about consumer preferences, such as the presence of a bequest motive, which prior studies have argued might rationalize low observed rates of annuitization. There continues to be debate over why real-world consumption profiles and annuity purchase decisions look the way they do. However, as we show, the implications for life-extension depend primarily on the real-world consumption profiles themselves, not the reasons that lie beneath.

Like prior studies on the value of life, we focus throughout this paper on the demand for health and longevity. Quantifying optimal health spending requires additionally modeling the supply of health care (Hall and Jones 2007). In light of all the variation in healthcare delivery systems, a wide variety of plausible approaches can be taken to this modeling problem, which we leave to future research.

II.A. The fully annuitized value of life

Let $c(t)$ be consumption at time t, W_0 be baseline wealth, $m(t)$ be exogenously determined income, ρ be the rate of time preference, and r be the rate of interest.[3] Let W be the net present value of wealth and future earnings at baseline. Finally, define $q(t)$ as health-related quality of life at time t. Since it sacrifices little generality in our application, we take $q(t)$ as exogenous. As needed, one can consider any relevant quality

[3] It is straightforward to incorporate endogenous labor supply (Murphy and Topel 2006). In the stochastic mortality model presented in Section III, we allow income to depend on the health state.

of life profile in concert with a given profile of mortality, and we investigate this issue in our empirical analysis later. The maximum lifespan of a consumer is T, and her mortality (hazard) rate at any point in time is given by $\mu(t)$, where $0 \le t \le T$. The probability that a consumer will be alive at time t is:

$$S(t) = \exp\left[-\int_0^t \mu(s)ds\right]$$

At time $t = 0$, the consumer fully annuitizes. We assume that annuitization is actuarially fair. The consumer's maximization problem is:

$$V(0) = \max_{c(t)} \int_0^T e^{-\rho t} S(t) u(c(t), q(t)) dt$$

subject to the budget constraint:

$$\int_0^T e^{-rt} S(t) c(t) dt = W = W_0 + \int_0^T e^{-rt} S(t) m(t) dt$$

The consumer's utility function, $u(c(t), q(t))$, depends on both consumption and health-related quality of life. We assume throughout that $u(\cdot)$ is strictly increasing and concave in its first argument, and twice continuously differentiable. Let $u_c(\cdot)$ denote the marginal utility of consumption. Associating the multiplier θ with the wealth constraint, optimal consumption is characterized by the first-order condition:

$$\frac{\partial V(0)}{\partial W} = \theta = e^{(r-\rho)t} u_c(c(t), q(t))$$

To analyze the value of life, let $\delta(t)$ be a perturbation on the mortality rate with $\int_0^T \delta(t)dt = 1$, and consider:

$$S^{\varepsilon}(t) = \exp\left[-\int_0^t (\mu(s) - \varepsilon\delta(s))ds\right], \varepsilon > 0$$

Let $c^{\varepsilon}(t)$ represent the equilibrium variation in $c(t)$ caused by this perturbation. As shown in Rosen (1988), the marginal utility of this life-extension is given by:

$$\begin{aligned}\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} &= \left.\frac{\partial}{\partial \varepsilon}\int_0^T e^{-\rho t} S^{\varepsilon}(t) u\big(c^{\varepsilon}(t), q(t)\big) dt\right|_{\varepsilon=0} \\ &= \int_0^T \left[e^{-\rho t} u(c(t), q(t)) + e^{-rt}\theta\big(m(t) - c(t)\big)\right]\left[\int_0^t \delta(s)ds\right] S(t) dt\end{aligned}$$

The marginal value of life-extension is equal to the marginal rate of substitution between longer life and wealth:

$$\left.\frac{\partial V/\partial\varepsilon}{\partial V/\partial W}\right|_{\varepsilon=0} = \int_0^T e^{-rt} S(t)\left(\frac{u(c(t),q(t))}{u_c(c(t),q(t))} + m(t) - c(t)\right)\left[\int_0^t \delta(s)ds\right]dt \tag{1}$$

The value of a life-year is the value of a one-period change in survival from the perspective of current time:

$$v(t) = \frac{u\big(c(t),q(t)\big)}{u_c\big(c(t),q(t)\big)} + m(t) - c(t) \tag{2}$$

The value of a life-year, $v(t)$, is equal to the value of consumption in that year plus net savings, $m(t) - c(t)$. The net savings term is a consequence of the requirement that annuities be actuarially fair. The value of a life-year can be rewritten as:

$$v(t) = m(t) + c(t)\left(\frac{u\big(c(t),q(t)\big)}{c(t)u_c\big(c(t),q(t)\big)} - 1\right) = m(t) + c(t)\phi(c,q)$$

where $\phi(c,q)$ represents the consumer surplus value per unit of consumption. It is positive if average utility exceeds marginal utility. A life-year thus adds value through two different channels: an increase in earnings, $m(t)$, which can finance additional consumption, and an increase in consumer surplus, $c(t)\phi(c,q)$.[4]

A canonical choice for $\delta(\cdot)$ in equation (1) is the Dirac delta function, so that the mortality rate is perturbed at $t = 0$ and remains unaffected otherwise. This then yields an expression that is commonly called the value of statistical life (VSL):

$$VSL = \int_0^T e^{-rt} S(t)v(t)dt \tag{3}$$

VSL corresponds to the value that the individual places on a marginal reduction in the risk of death in the current period. For example, it is the amount that 1,000 people are collectively willing to pay to eliminate a current risk that is expected to kill one of them. It is equal to the present discounted value of lifetime consumption, plus the change in net savings. Holding wealth constant, VSL increases with survival, which implies increasing returns in health improvements (Murphy and Topel 2006). Conversely, this leads to the conventional result that VSL falls when mortality rises.

VSL depends on how substitutable consumption is at different ages, i.e., on how easily an individual can reallocate consumption over time. Intuitively, if present consumption is a good substitute for future consumption, then living longer is less valuable. Define the elasticity of intertemporal substitution, σ, as:

$$\frac{1}{\sigma} \equiv -\frac{u_{cc}c}{u_c}$$

[4] Positive consumer surplus may require that consumption remain above a “subsistence” level, $\underline{c} > 0$.

In addition, define the elasticity of quality of life with respect to the marginal utility of consumption as:

$$\eta \equiv \frac{u_{cq} q}{u_c}$$

When η is positive, the marginal utility of consumption is higher in healthier states, and vice-versa. Taking logarithms of the first-order condition for consumption and differentiating with respect to time yields the rate of change for consumption over the life cycle:

$$\frac{\dot{c}}{c} = \sigma(r - \rho) + \sigma\eta \frac{\dot{q}}{q} \tag{4}$$

If one assumes that $r > \rho$, and that the marginal utility of consumption is higher when health status is better, then life-cycle consumption will have the inverted U-shape observed in real-world data.[5]

A crucial feature of the conventional model is that consumption growth over the life-cycle is independent of the mortality rate, because the individual is fully insured against longevity risk. This feature in turn implies that the rate of change in the value of a life-year is also not a function of the mortality rate:

$$\frac{\dot{v}}{v} = \left(\frac{1}{\sigma v}\frac{u}{u_c}\right)\frac{\dot{c}}{c} + \left(\frac{-\eta}{v}\frac{u}{u_c} + \frac{q}{v}\frac{u_q}{u_c}\right)\frac{\dot{q}}{q} + \frac{\dot{m}}{v}$$

In sum, we have identified two major features of the theory on the value of life under the conventional assumptions of full annuitization and deterministic mortality risk:

- The relative value of a life-year within a lifetime is independent of the mortality rate.
- The value of statistical life falls when mortality rises.

II.B. The uninsured value of life

Next, we consider a setting where the consumer lacks access to annuity markets and cannot borrow against future income. (We will consider various partial annuitization schemes in our empirical exercises.) To characterize this model without annuitization, we employ the Yaari (1965) model of consumption behavior under survival uncertainty. Let the state variable $W(t)$ represents current wealth at time t. The consumer's maximization problem is:

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[5] Under these assumptions, consumption will climb early in life as the benefits to savings diminish, and then decline later in life when quality of life deteriorates. This inverted U-shape for the age profile of consumption has been widely documented across different countries and goods (Carroll and Summers 1991; Banks, Blundell, and Tanner 1998; Fernandez-Villaverde and Krueger 2007).

$$V\big(0, W(0)\big) = \max_{c(t)} \int_0^T e^{-\rho t} S(t) u(c(t), q(t)) dt$$

subject to:

$$W(0) = W_0,$$
$$W(t) \geq 0, W(T) = 0,$$
$$\frac{\partial W(t)}{\partial t} = rW(t) + m(t) - c(t)$$

If the non-negative wealth constraint binds, then the solution to the consumer's problem is to set $c(t) = m(t)$. Otherwise, the solution is to maximize subject to the constraint on the law of motion for wealth. We focus here on the latter, nontrivial case.

Optimal consumption is again characterized by the first-order condition:

$$\frac{\partial V\big(0, W(0)\big)}{\partial W(0)} = \theta = e^{(r-\rho)t} S(t) u_c(c(t), q(t))$$

Unlike in the case of perfect markets, the survival function enters the consumer's first-order condition for consumption. Instead of setting the discounted marginal utility of consumption equal to the marginal utility of wealth, the consumer sets the *expected* discounted marginal utility of consumption at time t equal to the marginal utility of wealth. This shifts consumption to earlier ages in the life-cycle, which is rational because consumption allocated to later time periods will not be enjoyed in the event of an early death.

The expression for the marginal utility of life-extension is:

$$\begin{aligned}
\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} &= \left.\frac{\partial}{\partial \varepsilon} \int_0^T e^{-\rho t} S^{\varepsilon}(t) u\big(c^{\varepsilon}(t), q(t)\big) dt\right|_{\varepsilon=0} \\
&= \int_0^T e^{-\rho t} \left[\int_0^t \delta(s) ds\right] S(t) u(c(t), q(t)) dt + \int_0^T e^{-\rho t} S(t) u_c(c(t), q(t)) \left.\frac{\partial c^{\varepsilon}(t)}{\partial \varepsilon}\right|_{\varepsilon=0} dt \\
&= \int_0^T e^{-\rho t} \left[\int_0^t \delta(s) ds\right] S(t) u(c(t), q(t)) dt + \theta \frac{\partial}{\partial \varepsilon} \int_0^T e^{-rt} c^{\varepsilon}(t) dt \\
&= \int_0^T e^{-\rho t} \left[\int_0^t \delta(s) ds\right] S(t) u(c(t), q(t)) dt,
\end{aligned}$$

where the last equality follows from application of the budget constraint.[6]

[6] The budget constraint $W(T) = 0$ implies $\int_0^T e^{-rt} c^{\varepsilon}(t) dt = W_0 + \int_0^T e^{-rt} m(t) dt$, a value which does not depend on survival and thus is unaffected by life extension.

Dividing this result by the marginal utility of wealth, θ, then yields the marginal value of life-extension:

$$\left.\frac{\partial V/\partial \varepsilon}{\partial V/\partial W}\right|_{\varepsilon=0} = \int_0^T e^{-\rho t}\left[\int_0^t \delta(s)ds\right] S(t)\,\frac{u\big(c(t),q(t)\big)}{u_c\big(c(0),q(0)\big)}dt \tag{5}$$
$$= \int_0^T e^{-rt}\left[\int_0^t \delta(s)ds\right]\frac{u\big(c(t),q(t)\big)}{u_c\big(c(t),q(t)\big)}dt$$

In this setting, the value of a life-year from the perspective of current time is:

$$v(t) = \frac{u\big(c(t),q(t)\big)}{u_c\big(c(t),q(t)\big)} \tag{6}$$

When the consumer is uninsured, the value of a life-year depends only on the value of consumption. The net savings term is absent in equation (6) because life-extension has no effect on the consumer's budget constraint.[7]

Choosing again the Dirac delta function for $\delta(\cdot)$ yields an expression for VSL that differs from the perfect markets case:

$$VSL = \int_0^T e^{-rt} v(t)dt \tag{7}$$

The value of statistical life is proportional to (expected) lifetime utility, and inversely proportional to the marginal utility of consumption. It is well known that removing annuity markets lowers lifetime utility (Yaari 1965). As we show more formally below, removing these markets also shifts consumption to earlier ages, thereby lowering the marginal utility of consumption at earlier ages. When consumers shift consumption forward, near-term life-years rise in value but distant life-years fall in value. Thus, the net effect of annuity markets on VSL is in general ambiguous. Put differently, exposure to longevity risk does not necessarily lower VSL. In the next section, we will show that this basic insight extends to exposing a consumer to a longevity "shock." We emphasize that in both cases the ambiguity in the relationship between mortality shocks and VSL depends critically on the absence of full annuitization.

Unlike the perfect markets case, the life-cycle consumption profile of the non-annuitized individual depends explicitly on the mortality rate. Taking logarithms of the first-order condition for consumption and differentiating with respect to time yields:

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[7] Unless the consumer survives until period T, she will die with positive wealth. Although this remaining wealth has no value to an individual with no bequest motive, it has value to society. When calculating the *social* value of life-extension in the empirical exercises presented in Section IV.C., we account for the effect of increased longevity on bequests by including a net savings term, defined to be the expected increase in future earnings net of consumption, as in equation (2). This term reflects the external effect of increased longevity on society's aggregate wealth.

$$\frac{\dot{c}}{c} = \sigma(r - \rho) + \sigma\eta\frac{\dot{q}}{q} - \sigma\mu(t) \tag{8}$$

Comparing this result to the standard case, given by equation (4), reveals both similarities and differences. As in the standard, fully annuitized model, the non-annuitized consumption profile described by equation (8) changes shape when the rate of time preference is above or below the rate of interest and when the quality of life changes. Unlike in the standard model, the consumption profile here depends explicitly on the mortality rate, $\mu(t)$. Higher rates of mortality depress the rate of consumption growth over the life-cycle. This rate of growth is always higher in the fully annuitized case, in which the last term drops out of the consumption growth equation (8). Put another way, removing the annuity market "pulls consumption earlier" in the life-cycle.

An appealing feature of the uninsured model is that it generates an inverted U-shape for the profile of consumption under the natural assumptions that consumption is constrained by a low income early in life and shifted forward by high mortality later in life. One need not impose the ad hoc assumptions on the signs of $r - \rho$ or η that are necessary in the fully annuitized model (e.g., Murphy and Topel 2006).

The life-cycle profile of the value of a life-year in this uninsured setting is:

$$\frac{\dot{v}}{v} = \left(\frac{1}{\sigma} + \frac{c}{v}\right)\frac{\dot{c}}{c} + \left(\frac{qu_q}{u} - \eta\right)\frac{\dot{q}}{q} \tag{9}$$

An important implication of (9) is that willingness to pay for longevity depends on the life-cycle mortality profile because of its dependence on the rate of change in consumption, $\dot{c}/c$. Holding quality of life constant, it is evident from equation (6) that increases in the mortality rate—which shift consumption forward—will raise v, the current value of a life-year. Thus, mortality also shifts forward the value of life. All else equal, individuals who face poor survival prospects will pay more for a marginal (near-term) life-year, but less for a distant life-year, than healthy peers who face good survival prospects. This differs from the implications of the conventional model, in which higher mortality reduces the values of life-years but has no impact on their relative values.

At the aggregate level, as societies become richer and live longer, the fraction of wealth spent on health will depend not just on the income elasticity of health, but also on the degree of survival uncertainty they face. Furthermore, our results imply that public programs that increase annuitization rates, such as Social Security, will affect society's willingness to pay for longevity, thereby creating a feedback loop that could dampen or increase program expenditures.[8] In our empirical exercises, we will quantify how the degree of annuitization influences the value of statistical life.

To summarize, we have identified the following two properties of the uninsured model that contrast with those of the fully annuitized model:

- When mortality rises, near-term life-years rise in value, but distant life-years fall in value.
- The value of statistical life may rise or fall when mortality rises.

In the next section, we allow mortality to be stochastic so that we can investigate formally the effect of disease and other health shocks on the value of life. Before turning to that analysis, we pause to note that suffering a health shock is similar to removing access to annuity markets: both expose an individual to longevity risk. Not surprisingly, we shall see that health shocks also shift the value of life-years forward, with an ambiguous net effect on VSL.

III. STOCHASTIC MODEL

The previous analysis illustrates how relaxing the conventional assumption of full annuitization affects the relationship between mortality risk and the value of life. The conventional framework is ill-equipped to study the influence of mortality risk for another reason as well. Just like our deterministic model above, it treats the mortality rate as a nonrandom parameter. Thus, shifts in the mortality rate reflect preordained and anticipated changes in mortality. In the real world, however, neither the timing nor the size of shifts in the mortality rate is known. As a related matter, the conventional framework does not allow for different health states. This omission precludes a meaningful analysis of the value of preventing health deterioration.

This section extends our analysis to allow for stochastic health shocks. Specifically, we assume that the individual's mortality rate, quality of life, and income now depend on her health state. Let Y_t be a continuous-time Markov chain with finite state space $Y = \{1,2,\dots,n\}$. Denote the transition intensities by:

$$\lambda_{ij}(t) = \lim_{h\to 0}\frac{1}{h}\mathbb{P}[Y_{t+h} = j | Y_t = i], j \neq i,$$
$$\lambda_{ii}(t) = -\sum_{j\neq i}\lambda_{ij}(t)$$

The mortality rate at time t is defined as:

[8] Philipson and Becker (1998) make the important, but distinct, point that the moral hazard effects of public annuity programs also increase an individual's willingness to pay for longevity gains.

$$\mu(t) = \sum_{j=1}^{n} \overline{\mu}_j(t)\mathbf{1}\{Y_t = j\}$$

where $\left\{\overline{\mu}_j(t)\right\}$ is exogenous and $\mathbf{1}\{Y_t = j\}$ is an indicator variable equal to 1 if the individual is in state j at time t and 0 otherwise. For analytical convenience and without meaningful loss of generality, we assume that individuals can transition only to higher-numbered states, i.e., $\lambda_{ij}(t) = 0\ \forall j < i$, so that the probability that a consumer in state i at time 0 remains in state i at time t is equal to:[9]

$$\tilde{S}(i,t) = \exp\left[-\int_0^t \left(\overline{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s)\right) ds\right]$$

A complete annuities market allows the consumer to insure fully against longevity risk even when mortality is stochastic.[10] Appendix D provides a full derivation for a setting with complete markets and demonstrates that stochastic mortality, by itself, does not alter the theoretical predictions of the conventional (deterministic) model as long as one maintains the assumption of full annuitization. Appendix D also derives expressions for the value of preventing illness when the consumer is fully annuitized. We defer discussion of those results until later in this section.

Here, we focus on the uninsured case, where the consumer lacks access to annuity markets and cannot borrow against future income. The consumer's maximization problem is:

$$V(0, W_0, Y_0) = \max_{c(t)} \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) u\left(c(t), q_{Y_t}(t)\right) dt \middle| Y_0, W_0\right] \tag{10}$$

subject to:

$$\begin{aligned} W(0) &= W_0, \\ W(t) &\geq 0, W(T) = 0, \\ \frac{\partial W(t)}{\partial t} &= rW(t) + m_{Y_t}(t) - c(t) \end{aligned}$$

[9] That is, an individual can transition from state i to j, $i < j$, but not vice versa. This does not meaningfully limit the generality of our model, because one can always define a new state $k > j$ with properties identical to state i.

[10] Reichling and Smetters (2015) show that when annuity markets are incomplete, stochastic mortality and correlated medical costs can explain the puzzling observation that many households do not fully annuitize their wealth. They take the positive correlation between health shocks and medical spending as a given. Our study provides a demand-side reason *why* these two phenomena are positively correlated.

As in the deterministic model presented in Section II.B, we focus on the non-trivial case where the non-negative wealth constraint does not bind. Define the consumer's objective function at time t as:

$$J(t,W(t),i) = \mathbb{E}\left[\int_0^{T-t} e^{-\rho u}\exp\left\{-\int_0^u \mu(t+s)ds\right\} u\big(c(t+u), q_{Y_{t+u}}(t+u)\big)du \,\middle|\, Y_t = i, W(t)\right]$$

Define the optimal value function as:

$$V(t,W(t),i) = \max_{c(s),s\geq t}\{J(t,W(t),i)\}$$

subject to the wealth dynamics above. Under conventional regularity conditions, if V and its partial derivatives are continuous, then V satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\begin{aligned}\big(\rho+\overline{\mu}_i(t)\big)V(t,W(t),i) = \max_{c(t)}\Bigg\{& u\big(c(t),q_i(t)\big) + \frac{\partial V(t,W(t),i)}{\partial W(t)}[rW(t)+m_i(t)-c(t)] \\ &+ \frac{\partial V(t,W(t),i)}{\partial t} + \sum_{j>i}\lambda_{ij}(t)[V(t,W(t),j)-V(t,W(t),i)]\Bigg\}, i = 1,\dots,n\end{aligned} \quad (11)$$

where $c(t) = c(t,W(t),i)$ is the (optimal) rate of consumption. In order to apply our value of life analysis, we exploit recent advances in the systems and control literature. Parpas and Webster (2013) show that one can reformulate a stochastic finite-horizon optimization problem as a deterministic problem that takes $V(t,W(t),j), j \neq i$, as exogenous. More precisely, we focus on the path of Y that begins in state i and remains in state i until time t. We denote optimal consumption and wealth in that path by $c_i(t)$ and $W_i(t)$, respectively.[11] A key advantage of this method is that it allows us to apply the standard deterministic Pontryagin maximum principle and derive analytic expressions.

Lemma 1:

The optimal value function for $Y_0 = i$ and $W(0) = W_0$, $V(0,W_0,i)$, for the following deterministic optimization problem also satisfies the HJB given by (11), for each $i \in \{1,\dots,n\}$:

$$V(0,W_0,i) = \max_{c_i(t)}\left[\int_0^T e^{-\rho t}\tilde{S}(i,t)\left(u(c_i(t),q_i(t)) + \sum_{j>i}\lambda_{ij}(t)V(t,W_i(t),j)\right)dt\right] \quad (12)$$

[11] Consumption, $c(t)$, is a stochastic process. We occasionally denote it as $c(t,W(t),Y_t)$ to emphasize that it depends on the states $(t,W(t),Y_t)$. When we reformulate our stochastic problem as a deterministic problem and focus on a single path $Y_t = i$, consumption is no longer stochastic because there is no uncertainty in the development of health states. We emphasize this point in our notation here by writing consumption as $c_i(t)$, and wealth as $W_i(t)$.

subject to:

$$W_i(0) = W_0,$$
$$\frac{\partial W_i(t)}{\partial t} = rW_i(t) + m_i(t) - c_i(t),$$

where $V(t, W_i(t), j)$ are taken as exogenous.

Proof of Lemma 1: see Appendix A

Because the value function V in (12) satisfies the HJB given by (11), it must also be equal to the consumer's optimal value function (see Proposition 3.2.1, Bertsekas (2005)). The present value Hamiltonian corresponding to (12) is:

$$H\left(W_i(t), c_i(t), p_t^{(i)}\right) = e^{-\rho t}\tilde{S}(i,t)\left(u\left(c_i(t), q_i(t)\right) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j)\right) + p_t^{(i)}[rW_i(t) - c_i(t) + m_i(t)]$$

where $p_t^{(i)}$ is the costate variable for state i. The necessary costate equation is:

$$\dot{p}_t^{(i)} = -p_t^{(i)} r - e^{-\rho t}\tilde{S}(i,t) \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \tag{13}$$

The solution to the costate equation can be obtained using the variation of the constant method:

$$p_t^{(i)} = \left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s) \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds\right] e^{-rt} + \theta^{(i)} e^{-rt}$$

where $\theta^{(i)}$ is a constant. The necessary first-order condition for consumption is:

$$p_t^{(i)} = e^{-\rho t}\tilde{S}(i,t) u_c\left(c_i(t), q_i(t)\right) \tag{14}$$

where the marginal utility of wealth at time $t = 0$ is $\frac{\partial V(0, W_0, i)}{\partial W_0} = p_0^{(i)} = u_c\left(c_i(0), q_i(0)\right)$. Since the Hamiltonian is concave in c and linear in W, the necessary conditions for optimality are also sufficient (Seierstad and Sydsaeter 1977).

To analyze the value of life, we let $\delta(t)$ be a perturbation on the mortality rate in state i with $\int_0^T \delta(t) dt = 1$ and consider:

$$\tilde{S}^{\varepsilon}(i,t) = \exp\left[-\int_0^t \left(\overline{\mu}_i(s) - \varepsilon\delta(s)\right) + \sum_{j>i} \lambda_{ij}(s)\, ds\right], \text{where } \varepsilon > 0$$

We first derive an expression for the effect of this perturbation on expected lifetime utility.

Lemma 2:

The marginal utility of life extension in state i is equal to:

$$\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T \left[e^{-\rho t} \left(\int_0^t \delta(s) ds \right) \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t)\, V(t, W_i(t), j) \right) \right] dt$$

Proof of Lemma 2: see Appendix A

In order to facilitate comparison to the deterministic case, it is useful to derive an expression for the marginal utility of wealth at time t.

Lemma 3:

The expected marginal utility of wealth in state i at time t is equal to:

$$\frac{\partial V(t, W_i(t), i)}{\partial W_i(t)} = u_c\big(c_i(t), q_i(t)\big)$$
$$= \mathbb{E}\left[e^{(r-\rho)(\tau - t)} \exp\left\{ -\int_t^\tau \mu(s) ds \right\} u_c\big(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)\big) \middle| Y_t = i, W(t) = W_i(t) \right], \forall \tau > t$$

Proof of Lemma 3: see Appendix A

This is the stochastic analogue of the consumer's first-order condition from Section II.B, and it shows that the consumer sets the expected discounted marginal utility of consumption at time $\tau > t$ equal to the current marginal utility of wealth. Our next result demonstrates that the value of statistical life also takes the same basic form as in the deterministic case.

Proposition 4:

Set $\delta(\cdot)$ in the expression for the marginal utility of life-extension given in **Lemma 2** equal to the Dirac delta function. Dividing the result by the marginal utility of wealth at time $t = 0$ shows that VSL in state i is equal to:

$$VSL(i) = \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) \frac{u\left(c(t), q_{Y_t}(t)\right)}{u_c\left(c(0), q_{Y_0}(0)\right)} dt \middle| Y_0 = i, W(0) = W_0 \right] \qquad (15)$$

Applying **Lemma 3** and rearranging yields the following, equivalent expression for VSL in state i:

$$VSL(i) = \int_0^T e^{-rt} v(i,t) dt$$

where the value of a life-year, $v(i,t)$, is equal to the expected utility of consumption normalized by the expected marginal utility of consumption:

$$v(i,t) = \frac{\mathbb{E}\left[S(t)\, u\left(c(t), q_{Y_t}(t)\right)\middle| Y_0 = i, W(0) = W_0\right]}{\mathbb{E}\left[S(t)\, u_c\left(c(t), q_{Y_t}(t)\right)\middle| Y_0 = i, W(0) = W_0\right]}$$

Proof of Proposition 4: see Appendix A

Analogous to the earlier setting with deterministic mortality, the value of statistical life is proportional to the expected (lifetime) utility of consumption, and inversely proportional to the marginal utility of consumption. As we shall show below, a negative health shock increases current consumption, causing the net effect on VSL to be ambiguous. This parallels the result from Section II.B that removing access to annuitization, thereby exposing a consumer to longevity risk, has an ambiguous effect on VSL.

We can derive an expression for the life-cycle profile of consumption from (14), the first-order condition for consumption. Differentiating with respect to t, plugging in the result for the costate equation and its solution, and rearranging yields:

$$\frac{\dot{c}_i}{c_i} = \sigma(r-\rho) + \sigma\eta\frac{\dot{q}}{q} - \sigma\bar{\mu}_i(t) - \sigma\sum_{j>i}\lambda_{ij}(t)\left[1 - \frac{u_c\left(c(t, W_i(t), j), q_j(t)\right)}{u_c\left(c(t, W_i(t), i), q_i(t)\right)}\right] \quad (16)$$

As in the deterministic case, the rate of change is a declining function of the individual's current mortality rate, $\bar{\mu}_i(t)$: removing the annuity market "pulls consumption earlier" in the life-cycle. Unlike in the deterministic case, there is now an additional source of risk, captured by the fourth term in equation (16). This term represents the possibility that the consumer might transition to a different health state in the future. This transition would shift consumption further if the marginal utility of consumption in those states is likely to be low. As we show below, this is likely to be the case if mortality in those states is high.

Equation (16) describes consumption dynamics conditional on the individual's health state i. It is not readily apparent whether stochastic mortality *on average*, across all states, causes consumption to shift forward relative to deterministic mortality. That said, one should expect stochastic mortality to shift consumption forward by less than in the deterministic case. Intuitively, this is because a stochastic environment allows an individual to react to unanticipated health shocks by adjusting her consumption. Put differently, a deterministic model is equivalent to a stochastic model where the consumer is forced to keep consumption constant across states. Consumers prefer the ability to adjust consumption, so that they can consume less in healthy states and more in sick states. We have confirmed this intuition in (unreported) empirical exercises that assume CRRA utility: on net, stochastic mortality causes consumers to shift consumption forward a bit less than deterministic mortality.

What happens when an individual transitions to a new health state? Because the consumer is not fully annuitized, consumption will jump. The sign of the jump can in general be positive or negative, depending on the characteristics of the new health state relative to the old state. Because there is no consensus regarding the sign of health state dependence, $u_{cq}(\cdot)$, let alone its magnitude, we hold quality of life constant for the time being, and return to this issue in our empirical analysis.[12] We will also assume that the health shock does not decrease income or wealth. Under these two assumptions, the model predicts that transitioning to a state where current and future expected mortality are high will shift consumption forward, and vice versa. Our next result proves this formally for a two-state case.[13] Our empirical exercises explore the implications of health shocks that affect both mortality and quality of life, and that are accompanied by shocks to wealth – whether due to the burden of medical spending or to decreases in time available for work.

Proposition 5:

Let there be $n = 2$ states with identical quality of life profiles, so that $q_1(s) = q_2(s)\ \forall s$. Assume that the transition intensities $\lambda_{12}(s)$ are uniformly bounded (finite), and that $\overline{\mu}_1(s) < \overline{\mu}_2(s)\ \forall s$, so that state 1 is "healthy" and state 2 is "sick." Suppose that the consumer transitions from state 1 to state 2 at time t, with no accompanying decrease in income (i.e., $m_1(s) \le m_2(s)\ \forall s \ge t$). Then $c_1(t) \le c_2(t)$.

Proof of Proposition 5: see Appendix A

It follows immediately from **Proposition 5** that the value of near-term life-years will increase, and the value of distant life-years will decrease, when transitioning from a healthy state with low mortality to a sick state with higher mortality. Whether VSL rises or falls is ambiguous, however. A rise in mortality risk lowers lifetime utility, which reduces VSL, but it also reduces the marginal utility of consumption, which increases

[12] Viscusi and Evans (1990), Sloan et al. (1998), and Finkelstein, Luttmer, and Notowidigdo (2013) find evidence of negative state dependence. Lillard and Weiss (1997) and Edwards (2008) find evidence of positive state dependence. Evans and Viscusi (1991) find no evidence of state dependence. Murphy and Topel (2006) assume negative state dependence when performing their calibration exercises, while Hall and Jones (2007) assume state independence.

[13] The proof can be extended to allow for a larger number of states, but the conditions required to sign the jump in consumption then become a complicated function of the matrix of transition probabilities and state-specific mortality rates. The two-state case conveys the basic result without a meaningful loss of generality.

VSL. Thus, the net effect depends on the curvature of the utility function relative to the curvature of the marginal utility function.

We formally demonstrate this tradeoff by comparing the VSL of a (persistently) healthy individual to the VSL of an individual who just fell ill but is otherwise identical. To fix ideas, we focus on the effects of a single health shock and investigate how it affects VSL over the life-cycle. We know from **Proposition 5** that the sick person's optimal consumption is initially higher (see Figure 2). Under what conditions is the sick person's VSL also higher? To make headway we must introduce the notion of prudence. The elasticity of intertemporal substitution, σ, is a common measure of the utility curvature. Prudence is the analogous measure for the curvature of marginal utility (Kimball 1990). Define relative prudence as:

$$\pi \equiv -\frac{c u_{ccc}(\cdot)}{u_{cc}(\cdot)}$$

Our next result provides sufficient conditions for VSL to rise following an adverse shock to longevity.

Proposition 6:

Consider a two-state setting with assumptions set out in **Proposition 5**. Assume further that utility is positive and that preferences satisfy the additional condition:[14]

$$\pi < \frac{2}{\sigma} \tag{17}$$

Suppose that the consumer transitions from state 1 to state 2 at time t, and that $\lambda_{12}(\tau) = 0 \; \forall \tau > t$. Then $VSL(1,t) \leq VSL(2,t)$.

Proof of Proposition 6: see Appendix A

The condition (17) specified in **Proposition 6** is satisfied by many common preferences, such as CRRA with $\sigma < 1$ (which we employ in our empirical exercises) and quadratic preferences. Consumers with inelastic demand, i.e., preferences with a low value for σ, find it costly to reallocate consumption over time. They therefore have a high willingness to pay for life-extension and are more likely to exhibit a rise in VSL following an adverse mortality shock. Likewise, consumers with low levels of prudence, π, have nearly-linear marginal utility that decreases rapidly with consumption. This generates a high willingness to pay for life-extension following a shock that increases consumption.

[14] The preference conditions specified in **Proposition 5** can be weakened. See the proof for details.

III.A. The value of statistical illness

Unlike the deterministic model, the stochastic model permits us to investigate the value of avoiding transitions to other health states. This requires only a slight modification to the analysis presented above, and will result in a more general concept we term the *value of statistical illness*. With a slight abuse of notation, let state $N+1$ correspond to death, so that $V(t, W(t), N+1) = 0$. Let $\delta_{ij}(t)$, $j \leq N$, be a perturbation on the transition intensity, $\lambda_{ij}(t)$, and let $\delta_{i,N+1}(t)$ be a perturbation on the mortality rate, $\overline{\mu}_i(t)$, $i \leq N$, where $\sum_{j=i+1}^{N+1} \int_0^T \delta_{ij}(t)dt = 1$, and consider:

$$\tilde{S}^{\varepsilon}(i,t) = \exp\left[-\int_0^t \left(\overline{\mu}_i(s) - \varepsilon\delta_{i,N+1}(s)\right) + \sum_{j=i+1}^{N} \left(\lambda_{ij}(s) - \varepsilon\delta_{ij}(s)\right) ds\right], \text{where } \varepsilon > 0$$

Proposition 7:

The marginal utility of preventing an illness or death is given by:

$$\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T e^{-\rho t}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s)\, ds\right)\left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t)\, V(t, W_i(t), j)\right) - \sum_{j>i} \delta_{ij}(t)\, V(t, W_i(t), j)\right] dt$$

Proof of Proposition 7: see Appendix A

The value of preventing an illness or death is equal to the marginal rate of substitution between the transition perturbation and wealth:

$$\left.\frac{\partial V/\partial \varepsilon}{\partial V/\partial W}\right|_{\varepsilon=0} = \int_0^T \frac{e^{-\rho t}\tilde{S}(i,t)}{u_c\left(c_i(0), q_i(0)\right)}\left[\left(\int_0^t \sum_{j>i} \delta_{ij}(s)\, ds\right)\left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t)\, V(t, W_i(t), j)\right) - \sum_{j>i} \delta_{ij}(t)\, V(t, W_i(t), j)\right] dt$$

As before, it is helpful to choose the Dirac delta function for $\delta(\cdot)$, so that the probability is perturbed at $t = 0$ and remains unaffected otherwise. It is also helpful to consider a reduction in the transition probability for only one alternative state, j, so that $\delta_{ik}(t) = 0 \; \forall k \neq j$. Applying these two conditions then yields what we term the value of statistical illness, $VSI(i, j)$:

$$VSI(i,j) = \frac{V(0, W(0), i) - V(0, W(0), j)}{u_c\left(c_i(0), q_i(0)\right)} \tag{18}$$

$$= VSL(i) - VSL(j)\frac{u_c\left(c_j(0), q_j(0)\right)}{u_c\left(c_i(0), q_i(0)\right)}$$

The interpretation of VSI is analogous to VSL: it is the amount that 1,000 individuals are collectively willing to pay in order to eliminate a current disease risk that is expected to befall one of them. Note that if health state j corresponds to death, so that $VSL(j) = VSL(N+1) = 0$, then $VSI(i,j) = VSL(i)$. Thus, VSI is a generalization of VSL.

It is instructive to compare VSI for the uninsured consumer, given in (18), to VSI for a fully annuitized consumer, which we denote as VSI^*:[15]

$$VSI^*(i,j) = VSL^*(i) - VSL^*(j) \tag{19}$$

Equation (19) justifies the common cost-effectiveness practice of equating the values of prevention and treatment (Drummond et al. 2015). Under full annuitization, the value of a life-year is equal across health states (holding quality of life constant). Thus, equation (19) implies that prevention and treatment are equally valuable, as long as they add the same number of expected life-years.[16]

In contrast, equation (18) shows that removing access to annuity markets breaks this equivalence between treatment and prevention. VSI in this case is not equal to the simple difference in VSL between the healthy and sick states, because VSL in the sick state is valued from the perspective of the sick, who are likely to have a lower marginal utility of consumption due to their shorter life span. This leads to the natural hypothesis that whenever VSL rises following an illness, the value of treatments (VSL per life-year) will be higher than equivalent preventive care consumed prior to the illness (VSI per life-year). It is simple to prove this for the case where the illness reduces life expectancy by one-half or more (proof available upon request), and we conjecture that the hypothesis is true under far more general conditions. In the empirical exercises that we present later, we find that the value of treatment is higher in a real-world population.

This difference in the values of prevention and treatment hinges on the distinction between ex ante and ex post valuations. Prevention is necessarily an ex ante concept, but treatments can be valued ex ante or ex post. From an ex ante point of view, the difference between equally effective preventive care and treatment is trivial—it does not matter much whether an individual avoids a disease by getting vaccinated when healthy or by paying an insurance premium to gain access to a drug that instantly cures her when ill. Put differently, there is little meaningful difference between prevention and treatment in the long run.

[15] When the consumer is fully annuitized, the value of her annuity depends on her health state. In particular, if she purchases an annuity in state i and then later transitions to a worse health state j, causing her life expectancy to fall, then the value of her annuity will also fall. This technicality is not reflected in the notation for equation (19); see Appendix D for details and discussion.

[16] Consistent with our model, Rheinberger, Herrera-Araujo, and Hammitt (2016) point out that prevention can be more valuable (ex ante) than treatment for a highly lethal, but rare, disease, because a disease-specific mortality reduction in this case has a much smaller effect on *total* life-years gained than a reduction in disease incidence.

But as Keynes dryly noted, “in the long run, we are all dead.” In the short run, society includes people who suffer from diseases that lack effective treatment and therefore value new medical innovations from an ex post perspective. Medical research policy decisions made on behalf of society should account for the value that research generates for both healthy and sick individuals.

To summarize, the stochastic mortality model yields the following implications:

- All else equal, when an individual transitions to a higher mortality state, near-term life-years rise in value, and distant life-years fall in value.
- The value of statistical life may rise or fall when an individual transitions to a higher mortality state; if the individual’s demand is sufficiently inelastic, or insufficiently prudent, then it will rise.
- Therapies that increase survival by treating sick patients are not generally worth the same as preventives that add the same amount of life expectancy for healthy patients; if the individual’s demand is sufficiently inelastic, or insufficiently prudent, then treatments are more valuable.

IV. QUANTITATIVE ANALYSIS

This section demonstrates how the value of statistical life depends on an individual’s health history, and illustrates that, under typical consumer preferences, the willingness to pay for treatment exceeds the willingness to pay for prevention. We also measure the social value of gains to health and longevity and how that value depends on the level of annuitization.

Our empirical framework incorporates survival and health status uncertainty into a life-cycle model and is related to a number of papers that study the savings behavior of the elderly (Kotlikoff 1988; Palumbo 1999; De Nardi, French, and Jones 2010). These prior studies allow health to affect wealth accumulation by including two or three different health states in the model. By contrast, we allow mortality and quality of life to vary across 20 different health states.

IV.A. Framework

We employ the discrete time analogue of our model. There are n health states. Denote the transition probabilities between health states by:

$$p_{ij}(t) = \mathbb{P}[Y_{t+1} = j | Y_t = i]$$

As in the continuous time model, the mortality rate at time t, $d(t)$, depends on the individual’s health state:

$$d(t) = \sum_{j=1}^{n} \overline{d}_j(t) \mathbf{1}\{Y_t = j\}$$

where $\{\overline{d}_j(t)\}$ are given and $\mathbf{1}\{Y_t = j\}$ is an indicator variable equal to 1 if the individual is in state j at time t and 0 otherwise. The probability of surviving from time period t to time period s is denoted as $S_t(s)$, where:

$$S_t(t) = 1,$$
$$S_t(s) = S_t(s-1)\big(1 - d(s-1)\big), s > t$$

The survival probability is stochastic because it depends on the individual's health history. Let $c(t)$ and $W(t)$ denote consumption and wealth in period t, respectively. The individual's health state at time , Y_t, determines her quality of life, $q_{Y_t}(t)$. Let ρ denote the utility discount rate, and r the interest rate. Assume that in each period the consumer receives an exogenously determined income, $y(t)$, and that the maximum lifespan of a consumer is T (i.e., $d(T) = 1$). Our baseline model assumes there is no bequest motive, although we relax this assumption in a later exercise.

The consumer's maximization problem is:

$$\max_{c(t)} \mathbb{E}\left[\sum_{t=0}^{T} e^{-\rho t} S_0(t) u\left(c(t), q_{Y_t}(t)\right) \middle| Y_0, W_0\right]$$

subject to:

$$W(0) = W_0,$$
$$W(t) \geq 0,$$
$$W(t+1) = \big(W(t) + y(t) - c(t)\big)e^r$$

The individual's period income is equal to $y(t) = (1-\tau)m(t) + a(t)$, where $a(t)$ is nonwage defined-benefit income financed by an actuarially fair earnings tax, τ. We choose the individual's labor earnings, $m(t)$, to fit data on average life-cycle earnings as estimated by the Current Population Survey and the Health and Retirement Survey (see Appendix B1 for details). Our retirement policy exercises, described in detail later, consider different levels of generosity for $a(t)$.

Unless stated otherwise, we assume that $r = \rho = 0.03$ (Siegel 1992; Moore and Viscusi 1990). Finally, we assume that utility takes the following CRRA form:

$$u(c, q) = q\frac{c^{1-\gamma}}{1-\gamma} - \frac{\underline{c}^{1-\gamma}}{1-\gamma} \tag{20}$$

As discussed in Section III, there is no consensus regarding the sign or magnitude of health state dependence ($u_{cq}(\cdot)$). Here, we assume a multiplicative relationship where the marginal utility of consumption is higher when quality of life is high, and vice versa.

We have normalized the utility of death to zero in (20). The consumer receives positive utility if she consumes an amount greater than $\underline{c}$, which represents a subsistence level of consumption. Consuming an amount less than $\underline{c}$ generates utility that is worse than death. Although adding a constant to the utility function does not affect the solution to the consumer's maximization problem, this constant matters for the value of life.[17] We are unaware of any empirical evidence on the magnitude of $\underline{c}$, the subsistence level of consumption in the United States. We assume it is equal to \$5,000, which is in line with the parameterization employed in Murphy and Topel (2006).

The parameter γ is the inverse of the elasticity of intertemporal substitution, an important determinant of both the value of life and the value of annuitization. We follow Hall and Jones (2007) and set $\gamma = 2$ in our main specification.

We employ dynamic programming techniques to solve for the optimal consumption path. The value function is defined as:

$$V(t, w, i) = \max_{c(t)} \mathbb{E}\left[\sum_{s=t}^{T} e^{-\rho(s-t)} S_t(s) u(c(s), q_i(s)) \middle| Y_t = i, W(t) = w\right]$$

We then reformulate the optimization problem as a recursive Bellman equation:

$$V(t, w, i) = \max_{c(t)} \left[u\big(c(t)\big) + \frac{1 - \overline{d}_i(t)}{e^{\rho}} \sum_{j=1}^{N} p_{ij}(t) V\big(t+1, \big(w + y(t) - c(t)\big)e^{r}, j\big) \right]$$

After solving for the optimal consumption path, we use the analytical formulas derived in the previous sections to calculate the value of life. Complete details are provided in Appendix C.

We are aware that there is significant uncertainty among economists regarding the proper values of many of the parameters in our model. The goal of the subsequent analyses is to illustrate the economic significance of our insights when our model is applied to real-world data using reasonable parameterizations. In some analyses, we investigate the sensitivity of our results to alternative assumptions regarding the elasticity of intertemporal substitution, $1/\gamma$, and to the presence of a bequest motive. While the value of γ matters

[17] Rosen (1988) was the first to point out that the level of utility is an important determinant of the value of life. See also additional discussion on this point in Hall and Jones (2007).

greatly for the size of VSL, it does not have any qualitative effect on our findings regarding the determinants of VSL.

The remainder of this section reports results from two separate empirical exercises. The first exercise illustrates the novel implications of our framework by showing that the value of statistical life can increase following a health shock, and that the value of treatment generally exceeds the value of prevention. The second exercise illustrates the effect of different annuitization schemes on the social value of improvements in longevity.

IV.B. Stochastic health shocks and the value of life

Conventional economic theory conceives of VSL as depending primarily on age and income/wealth. Our general framework with stochastic mortality and incomplete annuitization implies instead a substantial amount of variability in VSL within these categories. For example, individuals who have experienced a recent negative mortality shock have systematically higher VSL, although this VSL premium decays over time. We use real-world data on mortality and quality of life to estimate the degree to which VSL varies within the traditional categories of age and income, and describe the factors explaining the variation. Later exercises also incorporate data on medical spending and allow for a bequest motive. We focus here on the private value of statistical life[18] and abstract from potential externalities, e.g., investments in disease-prevention that might benefit public health insurance programs or other members of society. We consider the social value of health improvements in our second set of exercises.

The data for this set of exercises are provided by the Future Elderly Model (FEM), a widely published microsimulation model that employs comprehensive, nationally representative data from a wide array of sources (Michaud et al. 2011; Goldman et al. 2005; Lakdawalla, Goldman, and Shang 2005; Goldman et al. 2009; Lakdawalla et al. 2009; Goldman et al. 2013; Michaud et al. 2012; Goldman et al. 2010). The model, which has been released into the public-domain, produces estimates of mortality, disease incidence, quality of life, and medical spending at the individual level for people over the age of 50 with different comorbid conditions.[19] The FEM accounts for six different chronic conditions (cancer, diabetes, heart disease, hypertension, chronic lung disease, and stroke) and six different impaired activities of daily living

[18] That is, our calculations in this section do not account for net savings, which will generally be negative for the elderly population we focus on here because expected future consumption is larger than future income (see also discussion in footnote 7). All else equal, ignoring net savings decreases the value of treatment relative to prevention: prevention is consumed by the healthy, who live longer than the sick and thus have larger expected future consumption, i.e., their (negative) net savings are larger in magnitude. Incorporating net savings would therefore only strengthen our results.

[19] Additional details about the FEM's methodology are provided in Appendix B2. A complete technical description is available at roybalhealthpolicy.usc.edu/fem/technical-specifications/.

(bathing, eating, dressing, walking, getting in or out of bed, and using the toilet). The FEM provides us with a widely published and well-validated tool that combines information from multiple nationally representative data sources, including the Health and Retirement Study, the Medical Expenditure Panel Survey (MEPS), the Panel Study of Income Dynamics, and the National Health Interview Survey. This provides a number of advantages for our study. For instance, while the HRS possesses a uniquely rich set of covariates on health and wealth, it lacks survey questions that would allow us to calculate quality of life using validated survey instruments. To solve this problem, the FEM weaves together validated quality of life estimates from the MEPS and maps them to the HRS using variables common to both databases.

We divide the health space within the FEM into $n = 20$ states. Each state corresponds to the number (0, 1, 2, 3 or more) of impaired activities of daily living (ADL) and the number (0, 1, 2, 3, 4 or more) of chronic conditions, for a total of $4 \times 5 = 20$ health states. Health states are ordered first by number of ADL's and then by number of chronic diseases, so that state 1 corresponds to 0 ADL's and 0 chronic conditions, state 2 corresponds to 0 ADL's and 1 chronic condition, and so on. For each health state and age, the FEM estimates the probability of dying and the probability of transitioning to each of the other health states in the next year. As in the theoretical model, individuals can transition only to higher-numbered states, i.e., $p_{ij}(t) = 0 \ \forall j < i$. In other words, all ADL's and chronic conditions are permanent. The FEM also estimates quality of life for each health state and age, as measured by the EuroQol five dimensions questionnaire (EQ-5D). These five dimensions are based on five survey questions that elicit the extent of a respondent's problems with mobility, self-care, daily activities, pain, and anxiety/depression. These questions are then weighted using stated preference data to compute the relative importance of each.[20] The result is a single quality of life measure, the EQ-5D, typically reported on a scale from zero to one.

Table 1 presents basic descriptive statistics for the data provided by the FEM model. Initial life expectancy at age 50 ranges from 30.4 years for a healthy individual in state 1 to 8.6 years for an ill individual in state 20. Quality of life, as measured by the EQ-5D index, ranges from 0.54 to 0.88 at age 50. Columns (7) and (8) of Table 1 report the probability that an individual exits her health state but remains alive, i.e., acquires at least one new ADL or chronic condition within the following year. Health states are relatively persistent, with exit rates never exceeding 15 percent. State 20 is an absorbing state with an exit rate of 0 percent.

The model employed in this section assumes that individuals lack access to annuity markets. (We consider partial annuitization scenarios in Section IV.C.) To abstract from wealth effects, we set total wealth equal

[20] The five dimensions of the EQ-5D are weighted using estimates from Shaw, Johnson, and Coons (2005). The specific process for estimating the quality of life score is explained in the FEM technical documentation, which can be found in the supplemental information appendix of Agus et al. (2016).

to what it would be under full annuitization. Mechanically, we set wealth equal to $807,604 and provide all of it to the individual at baseline (age 50). This value for wealth corresponds to the net present value of future earnings at age 50 plus savings at that age, as estimated by the retirement policy model we employ in the next section. This permits an analytical solution to the consumer's problem (see Appendix C2). It also avoids numerical precision error and speeds up the calculations, which is especially useful when performing the Monte Carlo simulations described below.[21]

We now turn to our results. If an individual never suffers a health shock, then her consumption and VSL will decline smoothly with age. However, the arrival of a health shock can increase VSL, sometimes substantially. Figure 3 displays consumption and VSL for an initially healthy individual who develops one ADL (health state 6) at age 60, and then two more ADLs plus two chronic conditions (health state 18) at age 70. The first shock reduces her life expectancy by 3.0 years and her quality of life by 0.06. The second one reduces her life expectancy by 6.7 years and her quality of life by 0.20. This causes discontinuous jumps in consumption at ages 60 and 70 as a result of these two negative health shocks. The first shock has a mild effect on the declining trend in VSL, but the second increases her VSL at age 70 by nearly 50 percent, from $2.9 million to $4.3 million. This jump is driven by the reduction in life expectancy and would remain large even if quality of life were held constant.

Individual-level shocks generate substantial variability in VSL in the aggregate. Figure 4 reports results from a Monte Carlo simulation of 10,000 life-cycle modeling exercises. At age 50, all individuals are identical and have a VSL of $5.9 million. As they age, some begin to suffer health shocks that, at least initially, increase their VSL. By age 60, the VSL inter-vigintile range spans $4.2 to $5.3 million. This dispersion is compressed towards the end of life, when mortality reaches 100 percent.

Next, we calculate the value of statistical illness (VSI) for different diseases. Column (4) of Table 2 reports VSI at age 50 from the perspective of a healthy individual. Each value represents the healthy individual's willingness to pay for a marginal, contemporaneous reduction in the probability of developing an illness corresponding to one of the 19 other health states. The values are inversely related to life expectancy in the sick state because it is more valuable to prevent the onset of a lethal disease than a mild one. The highest VSI is $3.5 million, which corresponds to the value of preventing the onset of a sick state with 3 ADL's and 4 chronic conditions (health state 20). The interpretation is analogous to VSL: it is the amount that 1,000 healthy individuals would collectively be willing to pay in order to reduce their risk of developing

[21] Relaxing these assumptions is possible (and available upon request), but will prohibit the calculation of an exact solution. Hubbard, Skinner, and Zeldes (1995) show that failing to include a "welfare floor" in the budget constraint causes life-cycle models to overestimate savings for low-income households. Our exercises model middle-income individuals, however, for whom this issue is less important.

this illness by 1/1000. This value remains below the healthy individual's VSL, which represents the willingness to pay to avoid the extreme "illness" of dying.

How does the value of prevention compare to the value of treatment? We investigate this question by normalizing VSL and VSI by the number of life-years saved. In contrast to the conventional (fully annuitized) framework, here the value of a life-year depends on the whether the individual is sick or healthy. Intuitively, health gains are worth more after health shocks than before them, because those shocks accelerate consumption and increase the value of life.

Table 2 illustrates this point. For example, column (5) reports that a 50-year-old with one chronic condition and no ADL's (health state 2) has a marginal willingness to pay of $228,000 per life-year for a treatment that extends her life. Column (6), by contrast, reveals that a healthy individual is only willing to pay $115,000 per life-year saved through preventing the onset of health state 2. In this case, treatment is twice as valuable as prevention. Column (6) of Table 1 shows that the value of life-years saved by treating an illness always exceeds the corresponding value gained by prevention that illness – by as much as a factor of 10, for the sickest state in our model.

Figure 5 displays these results graphically. It depicts how VSL and VSI vary across our health states, which are arrayed along the x-axis from longest to shortest life expectancy. The solid blue bars depict VSL per life-year and demonstrate that the value gained through treatment is monotonically higher for states with lower remaining life expectancy. The dotted red bars show the value per life-year gained by preventing each health state, from the perspective of a perfectly healthy person. For instance, the left-most dotted red bar reports the value of each life-year saved when a perfectly healthy consumer reduces the risk of entering the health state with 27.7 years of life expectancy. Notice that VSI is relatively stable across health states. This makes sense, because VSI is calculated from the fixed perspective of a perfectly healthy person; therefore, consumption profiles and the marginal utility of consumption remain stable. The minor variation across these health states in VSI per life-year is due primarily to differences in current and expected future quality of life.

These results help explain the low private willingness to invest in prevention (Dranove 1998; Pryor and Volpp 2018). Holding health gains fixed, individuals have weak incentives to invest in prevention. This wedge in the value of prevention versus treatment thus magnifies any external benefits of prevention that further separate the private and social willingness to pay for prevention.

Note, however, that the gap between the value of treatment and prevention narrows in the years following the diagnosis. Figure 6 compares the value of treatment for the consumer who suffered the second health shock depicted in Figure 3 to the value of prevention for a consumer who never suffered that second health shock. The value of treatment exceeds the value of prevention, but only for the first 10 years following the shock. After that point, the sick patient has spent down much of her wealth, which causes a significant reduction in her VSL, although we note that most patients will have died before reaching this point. (Life expectancy at age 70 for sick patients in this health state is 8.1 years.) This result implies that first-line therapies are more valuable than later-line therapies.

Next, we incorporate medical spending data from the FEM into our framework. Appendix Figure A1 reports average out-of-pocket medical spending for selected health states, by age. These data are comprehensive and include all inpatient, outpatient, prescription drug, and long-term care spending that is not paid for by insurance. Spending is higher in sicker health states, and—consistent with De Nardi, French, and Jones (2010)—increases greatly at older ages, when long-term care expenses arise.

Incorporating these spending data directly into our model would require numerical optimization methods. Instead, we reformulate these data as wealth shocks, which should yield qualitatively similar results while still allowing us to calculate an exact solution to the consumer's problem. This approach has the additional benefit of yielding insight into health shocks that reduce labor supply, since these would also ultimately reduce wealth. To implement, we modify the law of motion for wealth so that the individual's effective interest rate depends on her health state:

$$W(t+1) = \big(W(t) - c(t)\big)e^{r(t,Y_t)}$$

where $r(t, Y_t) = 0.03 + \ln[1 - s(t, Y_t)]$ and $s(t, Y_t)$ is the share of an individual's wealth spent on medical and nursing home care in health state Y_t and time t.[22] Appendix C2 provides additional details.

Figure 7 illustrates that incorporating medical spending reduces VSL slightly but does not otherwise appreciably alter its life-cycle profile, even in the presence of significant health shocks. This remains true even if we employ total, rather than out-of-pocket, medical spending. The reason is that the difference in medical spending between healthy and sick individuals is small relative to the variation in spending by age (see Appendix Figure A1). A sufficiently large idiosyncratic spending shock will have a significant impact, however. This is illustrated by the dotted black line in Figure 7, which plots VSL for a hypothetical case where the individual's wealth falls by 30 percent following the health shock at age 70, rather than by the much smaller medical spending amount estimated by the FEM. Although VSL still increases at age 70, the

[22] Specifically, we calculate $s(t, Y_t)$ by dividing out-of-pocket medical spending in health state Y_t at time t by $W(t)$, where $W(t)$ was estimated by our model for a healthy individual in a setting with no medical spending. Our results are similar if we instead use wealth estimates from the Health and Retirement Study.

rise is far smaller than in the other two cases. Thus, while accounting for typical medical spending does not appear to alter our basic results, catastrophic expenditures do matter.

Our last exercise values the longevity gains experienced over the past 15 years. During this period, all-cause mortality for the US population ages 50 and over has fallen by 18%, with cancer and heart disease mortality both falling by 21%.[23] Panel A of Table 3 values these health gains from the perspective of a current 50-year-old. In a setting with no out-of-pocket medical spending, the private value of the reduction in all-cause mortality is worth \$95,000 to \$302,000, depending on the assumed value of relative risk version. The values are reduced slightly if we include out-of-pocket medical spending. Panel B shows that these estimates are reduced by 10 to 20 percent if we incorporate a bequest motive into the model.[24] At the end of the next section, we discuss in more detail the relationship between bequest motives and the value of life.

IV.C. Retirement policy and the value of life

This section explores the link between retirement policy and the value of life. We build up to these results by calculating how the value of statistical life varies over the life-cycle under alternative annuitization policies. We then calculate how these alternative policies influence the value of permanent reductions in mortality. All our calculations account for the effect of mortality reduction on net savings, regardless of the degree of annuitization. This facilitates comparison across different annuitization scenarios and makes it appropriate to interpret our estimates as the social value of increased longevity. (See footnote 7.)

We initiate the model at age 20 and assume nobody survives past age 100. We obtain data on age-specific mortality rates from the Human Mortality Database. Because these mortality data are not available by health state, in this section we will assume deterministic mortality. (This corresponds to specifying $n = 1$ health states.) In addition, we abstract from the role of quality of life by setting $q(t) = 1$, because aggregate, nationally representative data on quality-of-life trends are not generally available.

Unlike in the previous section, the individual receives a flow of income instead of a baseline endowment of wealth. This feature is important here, because it allows us to model the effects of retirement and annuitization. Moreover, it is computationally simple to incorporate into this model, because we have only a single health state to contend with. Recall that the individual's period income is equal to $y(t) = (1 - \tau)m(t) + a(t)$, where $a(t)$ is nonwage defined-benefit income financed by an earnings tax, τ. We

[23] Source: authors' calculations using mortality data from the National Vital Statistics.

[24] We follow Fischer (1973) and allow the bequest motive takes a CRRA form, which allows us to again calculate an exact solution to the consumer's problem. See Appendix C2 for details.

consider three different policy scenarios in the main text. In the first, financial markets are absent, and the consumer's income equals her labor earnings: $y^1(t) = m(t)$. Thus, her consumption is limited by current period income and savings from prior periods. The second scenario introduces an actuarially fair Social Security program that provides an annuity equal to \$16,195 beginning at age 65.[25] In this second scenario, the consumer is partially annuitized, but she still lacks access to financial markets and cannot borrow against her future income. The third scenario increases the size of the Social Security pension by 50 percent. Finally, in the appendix we also present results for the case where the consumer fully annuitizes at age 20 and enjoys a constant annuity stream, $\bar{y} = \bar{a}$, provided by an actuarially fair and complete annuities market. The income streams in all scenarios are related according to the following equation:

$$\sum_{t=1}^{T} \frac{y^1(t)S(t)}{e^{r(t-1)}} = \sum_{t=1}^{T} \frac{y^2(t)S(t)}{e^{r(t-1)}} = \sum_{t=1}^{T} \frac{y^3(t)S(t)}{e^{r(t-1)}} = \bar{y} \sum_{t=1}^{T} \frac{S(t)}{e^{r(t-1)}}$$

Our assumed interest rate of 3 percent and our data on mortality and earnings imply a full annuity value of $\bar{y} = \$37{,}897$.

The life-cycle profiles of consumption for the first two policy scenarios are displayed in Figure 8. Consumption is constrained by the consumer's low income in early life. She saves during middle age when income is high, and then consumes her savings during retirement until eventually her consumption equals her pension (if available). Consumption for an individual with no annuity is "shifted forward" relative to an individual with a Social Security pension. This effect is particularly dramatic in the final 10 years of life, when old consumers outlive their wealth. This is not surprising: a primary benefit of an annuity is its ability to provide income to consumers in their oldest ages.

Figure 9 shows that this difference in consumption generates a corresponding difference in the value of a life-year. Individuals place a low value on life-years at very young and very old ages, because consumption is low. The slight drop at age 65 reflects the effect of retirement on the net savings component of the value of life.

Figure 10 displays the corresponding value of statistical life (VSL) for these two scenarios, as calculated by equation (7). At age 40, VSL is equal to \$7 million for an individual with no annuity, and \$8 million for an individual who will be eligible for Social Security at age 65. Both these values are within the ranges estimated by empirical studies of VSL for working-age individuals (see O'Brien 2018 for a recent review). Figure 10 also shows that VSL is greater at older ages for a person with a Social Security pension than it is

[25] This corresponds to the average retirement benefit paid by Social Security to retired workers in 2016 (www.ssa.gov/policy/docs/quickfacts/stat_snapshot/2016-07.pdf).

for a person with no annuity. This suggests that annuity programs are complementary with retiree healthcare programs and other investments in life-extension for the elderly population.

Finally, we calculate the value of historical reductions in mortality for these different annuitization scenarios, as well as the prospective value of permanent reductions in future mortality for selected diseases. Let δ denote a vector of mortality reductions for different ages. As in Murphy and Topel (2006), we calculate the total social value of a mortality reduction by aggregating over the age distribution of the 2015 US population:

$$Social\ Value = \sum_{a=0}^{110} VLE(a, \delta) f(a)$$

where $VLE(a, \delta)$ is defined as in equation (5), and $f(a)$ is the count of people alive in 2015 at age a.[26]

We report our results in Table 4. Life expectancy at birth increased by over 10 years between 1940 and 2010. Like Murphy and Topel (2006), we find that the social value of these past longevity gains is substantial: the post-1940 gains are worth over $100 trillion today, and the post-1970 gains are worth over $50 trillion. Comparing results for different annuitization scenarios informs our understanding of the interaction between retirement policies and the value of longevity. For example, consider the introduction of Social Security over the last century. Comparing column (1) to column (2) of Table 4 suggests that this increased the value of post-1940 longevity gains by $11.5 trillion (10.5 percent) and increased the value of post-1970 gains by $6.2 trillion (11.6 percent). One way to interpret these values is to compare them to the longevity insurance value of Social Security, which is approximately $17 trillion.[27] Thus, the interaction between post-1940 longevity gains and Social Security is worth about half as much as the longevity insurance value of the entire Social Security program itself.

Table 4 also reveals that Social Security has raised the value of a 10 percent cancer mortality reduction by $427 billion, or 13 percent. Alternatively, it has raised the value of a 10 percent reduction in all-cause mortality by $1.38 trillion (12 percent). Column (3) reports that increasing the size of Social Security pensions by 50 percent would add $723 billion more to that value.

[26] Specifically, $VLE(a, \delta) = \int_a^{100} e^{-r(t-a)} \left[\int_a^t \delta(s) ds\right] v(t) dt$. We assume $VLE(a, \delta) = VLE(20, \delta)$ for $a < 20$, and equal to $VLE(100, \delta)$ for $a > 100$. Unlike Murphy and Topel (2006), our social value calculation does not account for the value that mortality reductions generate for future (unborn) populations.

[27] This value is calculated using the methodology of Mitchell et al. (1999) and does not account for other potential benefits of Social Security such as protection against inflation risk. See Appendix C1 for details.

A bequest motive encourages individuals to delay consumption, because money saved for consumption in old age also has the added benefit of increasing bequests in the event of death. Its effects on consumption and the value of longevity are therefore similar to that of increased annuitization. Since bequests are much more common among the wealthiest consumers (Hurd and Smith 2002), they are unlikely to matter much for our main estimates, which pertain to the median individual. However, for illustrative purposes we have also estimated our main specification under the assumption of a strong bequest motive that significantly affects savings behavior even for the median individual.[28] Those results, illustrated in Figure 11, demonstrate that a bequest motive lowers the value of statistical life prior to age 65, and increases it at older ages. Intuitively, bequest motives increase the value of saving at younger ages. Appendix Table A3 further shows that in this case, the effect of Social Security on the value of post-1940 longevity gains is $5.5 trillion (5.1 percent), or about half as large as in a setting with no bequest motive. This suggests that the effect of retirement policy on the value of life matters most for non-wealthy individuals, whom are less likely to have a significant bequest motive.

To summarize, our model predicts that annuitization raises the value of life for the elderly. This should cause them to spend more on healthcare and invest more in healthy behaviors, which in turn should ultimately manifest in increased life expectancy. This dovetails with the point, made by Philipson and Becker (1998), that the moral hazard effects of retirement programs also increase the willingness to pay for longevity. Philipson and Becker (1998) analyze data from Virga (1996) and find that people with more generous annuities live longer than those with less generous annuities. They interpret this as the effect of endogenous longevity investments, which are encouraged among highly annuitized individuals who do not bear the full cost of an increase in their longevity. In our model, by contrast, annuitization increases the value of life even when annuities are actuarially fair, because they protect against the risk of outliving one's wealth. Given that these effects reinforce each other, it is not surprising that increases in the generosity of public pensions in developed countries have been accompanied by large increases in public spending on retiree healthcare.

V. CONCLUSION

The economic theory surrounding the value of life has many important applications. Yet, like most theories, it suffers from several anomalies that appear at odds with intuition or empirical facts – e.g., the apparent preferences of consumers to pay more for life-extension when survival prospects are bleaker. We have demonstrated that several of these anomalies can be reconciled without abandoning the standard

[28] When accounting for a bequest motive in this exercise, we follow Kopczuk and Lupton (2007) and assume the utility from leaving a bequest is linear in wealth. See Appendix C1 for details.

framework, simply by relaxing its strong assumptions about full annuitization and deterministic mortality. Moreover, relaxing these assumptions generates new predictions with implications for health policy and behavior. We show that VSL varies with the arrival of mortality shocks, and that a given gain in longevity can be more valuable to a consumer who has less life remaining, and vice-versa. Even holding wealth and income fixed, VSL may vary by $1 million or more for a 50-year-old. In addition, we demonstrate an interaction between retirement policy and health policy: increases in annuitization significantly increase the value of life among the elderly. For instance, the US Social Security program has increased the value of mortality reductions, adding nearly $150 billion to the value of a 1 percent mortality decline.

Our findings have several implications for the valuation of health investments and for policy more generally. We show that the value of a life-year varies across types of risk, not just across types of people. It can be more valuable to add one month of life for a patient facing a highly fatal disease than for one facing a much milder ailment. Thus, health spending should be more targeted towards the severely ill than current economic models of cost-effectiveness suggest.

In addition, public programs that expand the market for annuities might simultaneously boost the demand for life-extending technologies. Intuitively, annuities calm consumer fears about outliving their wealth and thus enable more aggressive investments in life-extension. Viewed differently, our results also show that market failures in annuities affect the value of statistical life, and thus the socially optimal level of health care spending. This suggests that researchers and policymakers should pay more attention to the public finance interactions between pension and healthcare systems.

Finally, our framework offers a single unified framework for valuing both treatment and prevention. This provides a more practical tool for policymakers and decision makers, since many health investments involve preventing the deterioration of health, not a direct and immediate mortality risk. Our result that treatment can be more valuable to individuals than equally effective preventive care also provides one explanation for why it has proven to be so difficult for policymakers and public health advocates to encourage investments in the prevention of disease. Kremer and Snyder (2015) show that heterogeneity in consumer values distorts R&D incentives by allowing firms to extract more consumer surplus from treatments than with preventives. Our results suggest that differences in private VSL may reinforce this result and further disadvantage incentives to develop preventives.

Our analysis raises a number of important questions for further research. First, how does the theory change if we endogenize the demand for health and longevity? Elsewhere, we have studied how incomplete health insurance enhances the value of medical technology that improves quality of life, because such technology acts as insurance by compressing the difference in utility between the sick and healthy states (Lakdawalla, Malani, and Reif 2017). Less clear is how demands for the quantity and quality of life interact with financial market incompleteness of various kinds. Second, what are the most practical strategies for incorporating these insights into the literature on cost-effectiveness of alternative medical technologies? This literature typically assumes that life-years, or quality-adjusted life-years, possess a constant value. While flawed, this approach remains simpler to implement. Future research should focus on practical strategies for aligning cost-effectiveness analyses with the generalized theory of the value of life. Finally, what are the implications for the empirical literature on VSL? Empirical analysis has typically proceeded under the assumption that different kinds of mortality risk are all valued the same way, as long as they imply similar changes in the probability of dying (Viscusi and Aldy 2003; Hirth et al. 2000; Mrozek and Taylor 2002). Our framework casts doubt on this assumption and suggests the need for a more nuanced empirical approach. This missing insight may be one reason for the widely disparate empirical estimates of the value of statistical life.

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VII. TABLES AND FIGURES

Table 1. Summary statistics for the data employed in the life-cycle modeling exercise, by health state

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
			Life expectancy		Quality of life		Exit probability	
Health state	ADL's	Chronic conditions	Age 50	Age 70	Age 50	Age 70	Age 50	Age 70
1 (healthy)	0	0	30.4	14.0	0.884	0.873	4.2%	12.6%
2	0	1	27.7	12.4	0.850	0.840	3.6%	10.8%
3	0	2	24.1	10.4	0.812	0.804	3.6%	10.2%
4	0	3	20.0	8.4	0.773	0.765	3.9%	10.2%
5	0	4+	15.6	6.6	0.730	0.720	3.9%	7.9%
6	1	0	26.1	12.0	0.830	0.816	6.3%	14.7%
7	1	1	23.5	10.6	0.795	0.783	5.7%	12.7%
8	1	2	20.0	8.8	0.754	0.745	6.1%	12.2%
9	1	3	16.3	7.1	0.716	0.707	6.4%	11.7%
10	1	4+	12.7	5.5	0.669	0.662	6.1%	8.6%
11	2	0	23.8	10.8	0.781	0.765	7.3%	14.3%
12	2	1	21.0	9.4	0.746	0.731	7.5%	14.3%
13	2	2	17.6	7.8	0.706	0.693	7.5%	13.8%
14	2	3	14.5	6.3	0.669	0.655	7.5%	13.1%
15	2	4+	11.0	4.8	0.630	0.610	7.3%	10.6%
16	3+	0	21.4	8.9	0.700	0.692	3.4%	11.1%
17	3+	1	18.5	7.9	0.664	0.660	2.8%	8.5%
18	3+	2	15.2	6.4	0.622	0.622	2.3%	7.1%
19	3+	3	12.2	5.0	0.584	0.584	1.4%	5.3%
20	3+	4+	8.6	3.8	0.536	0.540	0.0%	0.0%

Notes: This table reports selected summary statistics for the data employed by the stochastic life-cycle modeling exercises presented in Section IV.B. Columns (1) and (2) report the number of impaired activities of daily living (ADL) and the number of chronic conditions, which together define a health state. Column (3)-(6) report life expectancy (in years) and quality of life for an individual in one of these health states. Quality of life is measured using the EQ-5D index, which ranges from 0 (death) to 1 (perfectly healthy). Columns (7) and (8) report the probability that an individual transitions to a different health state in the following year. All ADL's and chronic conditions are permanent, so individuals can transition only to higher-numbered health states. These microsimulation data were generated by the Future Elderly Model. More information about that model is available in Appendix B2.

Table 2. Per capita private value of medical treatment and preventive care at age 50, by health state (thousands of dollars)

(1)	(2)	(3)	(4)	(5)	(6)	(7)
				Willingness to pay per life-year		
Health state	Life expectancy	VSL	VSI	Treatment	Prevention	Treatment/Prevention
1 (healthy)	30.4	$5,878	N/A	$193	N/A	N/A
2	27.7	$6,302	$312	$228	$115	1.97
6	26.1	$6,786	$483	$260	$113	2.29
3	24.1	$6,930	$774	$288	$123	2.34
11	23.8	$7,421	$783	$312	$119	2.62
7	23.5	$7,321	$833	$312	$121	2.58
16	21.4	$8,021	$1,163	$375	$129	2.91
12	21.0	$8,089	$1,200	$386	$127	3.04
4	20.0	$7,780	$1,366	$388	$132	2.95
8	20.0	$8,151	$1,354	$408	$130	3.15
17	18.5	$8,782	$1,621	$476	$136	3.50
13	17.6	$9,057	$1,721	$514	$135	3.81
9	16.3	$9,248	$1,941	$566	$138	4.10
5	15.6	$8,966	$2,102	$575	$142	4.04
18	15.2	$9,949	$2,165	$655	$142	4.59
14	14.5	$10,308	$2,258	$712	$142	5.02
10	12.7	$10,771	$2,595	$846	$147	5.75
19	12.2	$11,468	$2,721	$943	$149	6.32
15	11.0	$12,081	$2,944	$1,102	$152	7.27
20	8.6	$13,988	$3,453	$1,621	$159	10.22

Notes: This table displays values (in thousands of dollars) produced by the stochastic life-cycle modeling exercise presented in Section IV.B. Values are sorted by life expectancy at age 50, as reported in column (2). Column (3) reports the value of statistical life (VSL) for a 50-year-old in each health state. Column (4) reports the values of statistical illness (VSI) from the perspective of a healthy individual in state 1, and can be interpreted as a healthy individual's willingness to pay (WTP) to prevent a marginal increase in the probability of transitioning to the health state specified in column (1). Column (5) reports a sick individual's WTP per life-year for a therapeutic treatment, which is equal to the value in column (3) divided by the value in column (2). Column (6) reports the healthy individual's corresponding WTP for preventive care, which is equal to the value in column (4) divided by the difference between 30.4 (life expectancy when healthy) and the value in column (2). Column (7) reports the ratio of the values reported in columns (5) and (6). The twenty health states listed in column (1) are defined in Table 1.

Table 3. Per capita private value of historical 2001-2015 health gains, at age 50 (thousands of dollars)

Disease	Increase in life expectancy at age 50 (years)	(1)	(2)	(3)	(4)	(5)	(6)
A. No bequest motive							
All causes	1.43	$95	$159	$302	$87	$142	$263
Cancer	0.39	$23	$40	$77	$21	$34	$65
Heart disease	1.21	$68	$116	$224	$59	$96	$185
B. Bequest motive							
All causes	1.43	$87	$143	$275	$75	$121	$225
Cancer	0.39	$22	$36	$70	$18	$29	$55
Heart disease	1.21	$66	$106	$204	$52	$78	$150
Relative risk aversion		1.5	2	2.5	1.5	2	2.5
OOP medical spending		No	No	No	Yes	Yes	Yes

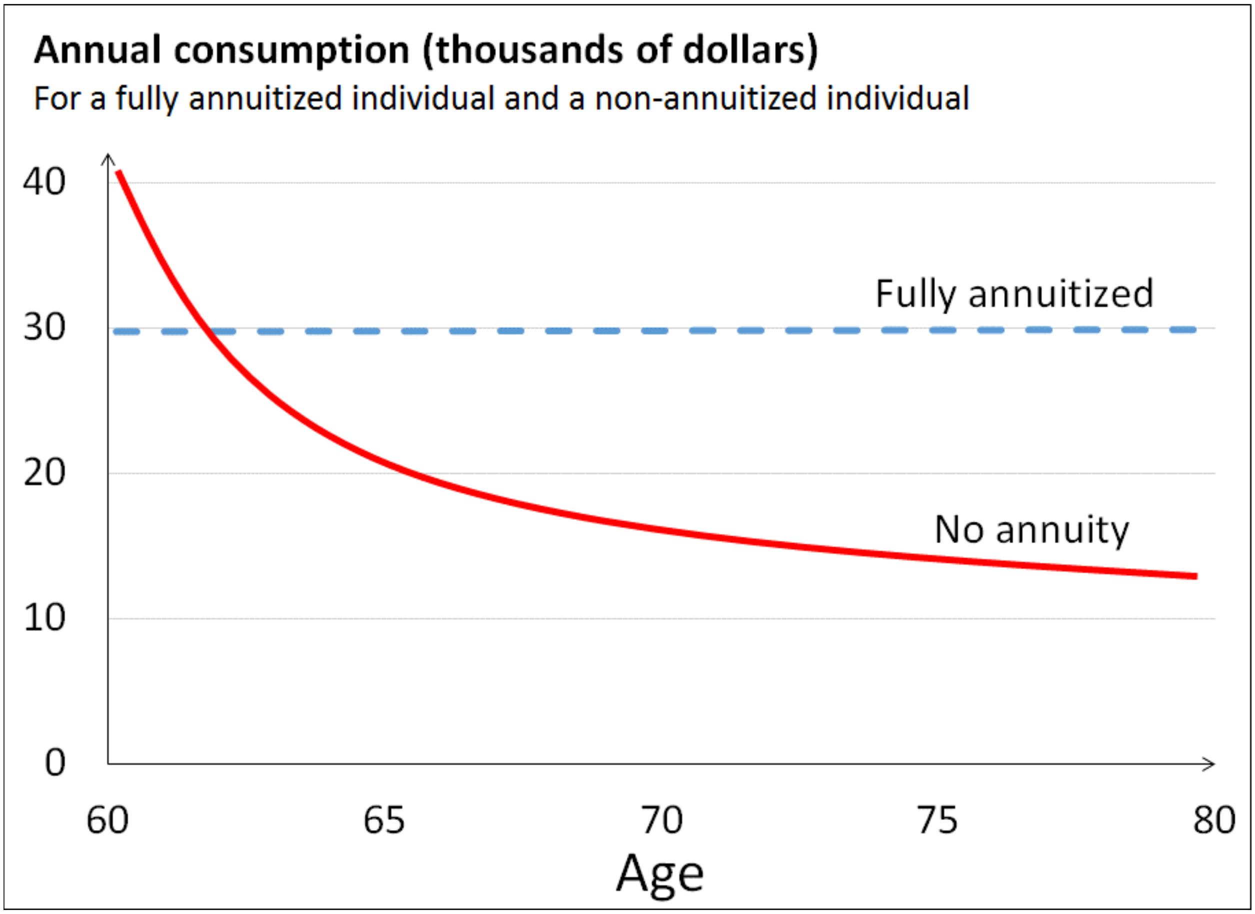
Notes: This table reports the value of the reduction in mortality experienced in the United States between 2001 and 2015, from the perspective of a 50-year-old alive in 2015. The cancer and heart disease calculations do not account for competing risks, and thus should be interpreted as holding mortality from all other causes constant. Columns (1)-(3) report results under the assumption that the individual has no out-of-pocket healthcare or nursing home costs. Columns (4)-(6) report results under the assumption that the health shocks are accompanied by an increase in healthcare and nursing home costs. The values in Panel A are calculated under the assumption that individuals do not have a bequest motive, while those in Panel B assume the bequest motive specification described in Appendix C2. The table also shows that these values increase with the size of the coefficient of relative risk aversion, which in our utility specification is equal to the inverse of the elasticity of intertemporal substitution.

Table 4. Aggregate social value of historical and prospective reductions in mortality (billions of dollars)

	(1)	(2)	(3)
	No annuity	**Social Security**	**Social Security + 50%**
A. Historical reduction			
1940-2010	$109,356	$120,855	$126,488
1970-2010	$53,492	$59,673	$62,769
B. 10% reduction, all ages			
All causes	$11,550	$12,928	$13,651
Cancer	$3,348	$3,775	$3,995
Diabetes	$368	$414	$437
Heart disease	$2,425	$2,744	$2,916
Homicide	$105	$102	$99
Infectious diseases	$166	$188	$201

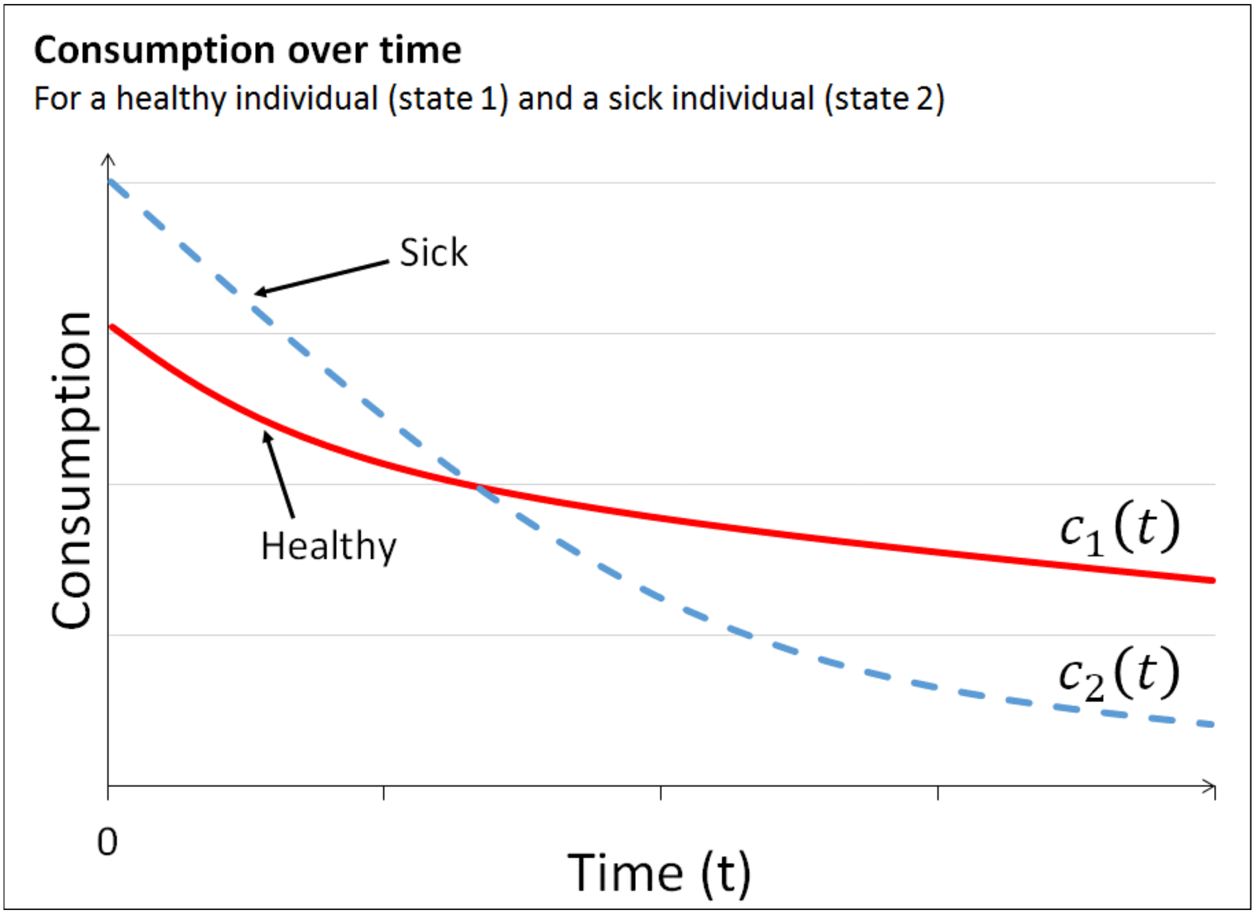
Notes: These aggregate values were calculated using the 2015 US population by age. Panel A reports the current value of historical reductions in all-cause mortality. Panel B reports the value of a 10 percent prospective reduction in mortality. Column (1) presents estimates under the assumption that individuals have no annuities. Column (2) presents estimates under the assumption that individuals receive typical Social Security benefits that are financed by an earnings tax. Column (3) increases the generosity of Social Security by 50%, financed by an increase in the earnings tax. The net present value of individuals' wealth at age 20 is the same across all three columns.

Figure 1. Illustrative example: annual consumption for fully annuitized and non-annuitized consumers



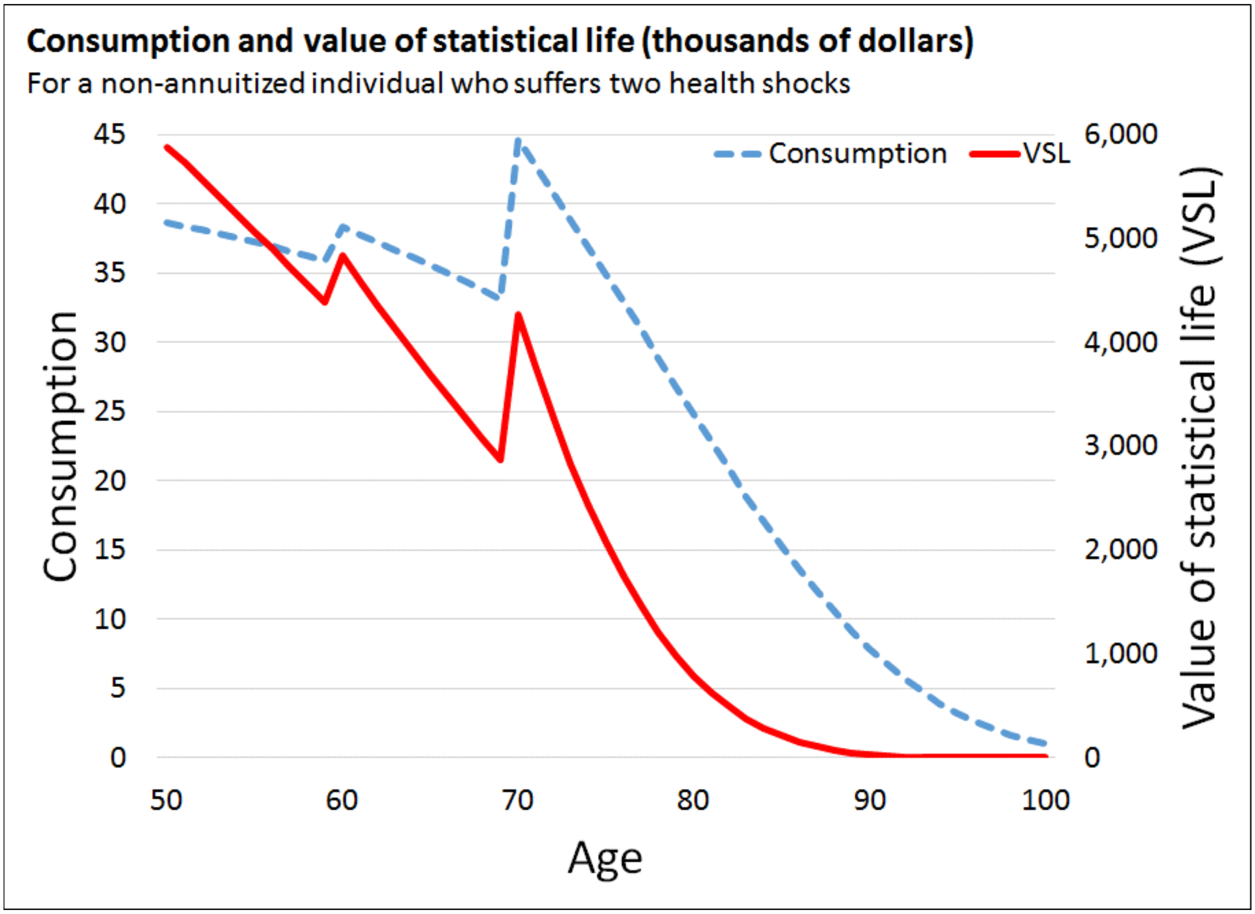
Notes: This figure illustrates the well-known result that it is optimal for a non-annuitized consumer who is exposed to longevity risk to shift her consumption forward in time, relative to a fully annuitized consumer. For simplicity, this example assumes that the optimal consumption profile of the fully annuitized consumer is flat.

Figure 2. Illustrative example: upon falling ill, consumption initially increases



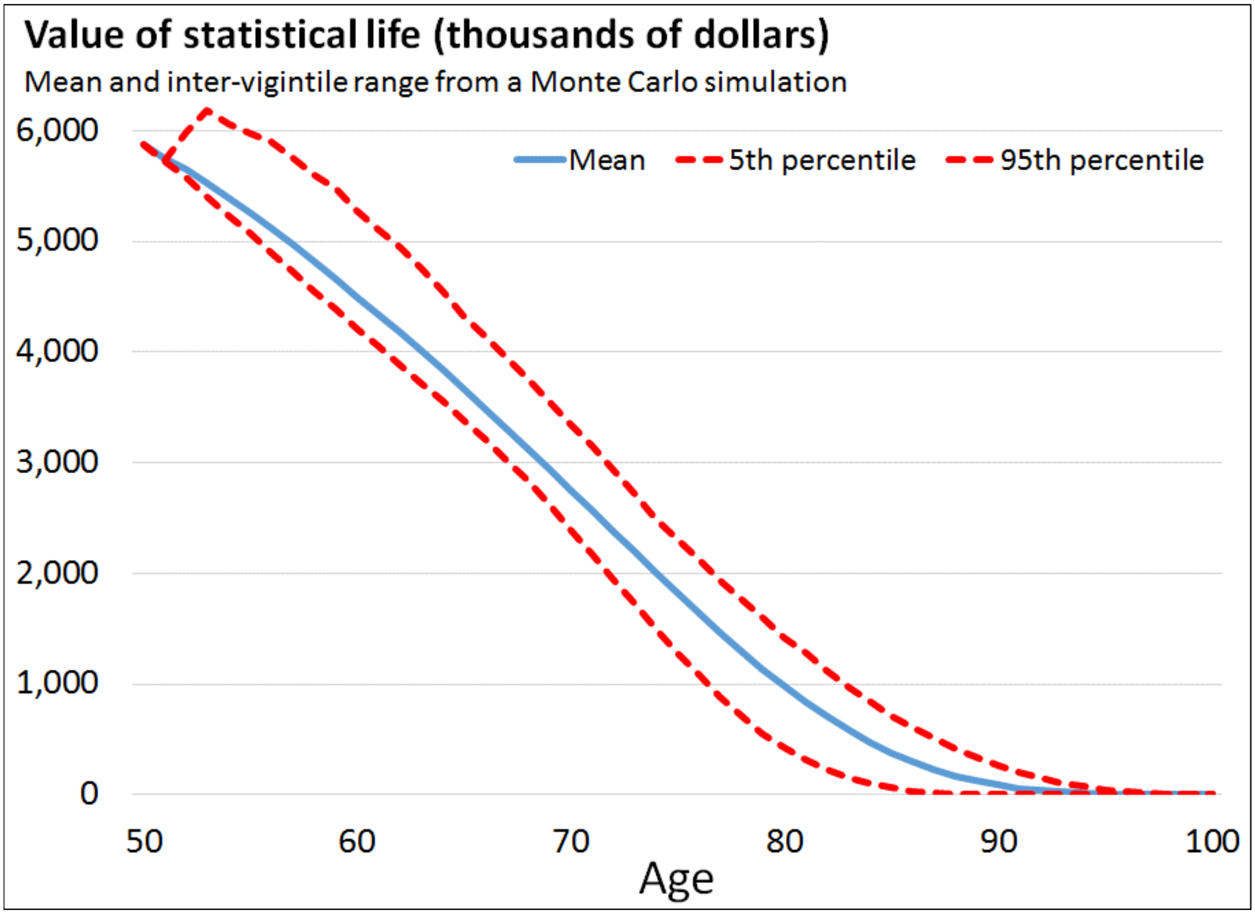
Notes: Both individuals have identical initial wealth at time $t = 0$. It is optimal for the sick individual (state 2) to consume at a higher rate than the healthy individual (state 1) because she has lower life expectancy. Thus, initial consumption at time $t = 0$ in the sick state is higher than in the healthy state, i.e., $c_2(0) > c_1(0)$. **Proposition 6** provides conditions under which VSL at time $t = 0$ in the sick state is also higher.

Figure 3. Consumption and the value of statistical life can increase when an individual falls ill



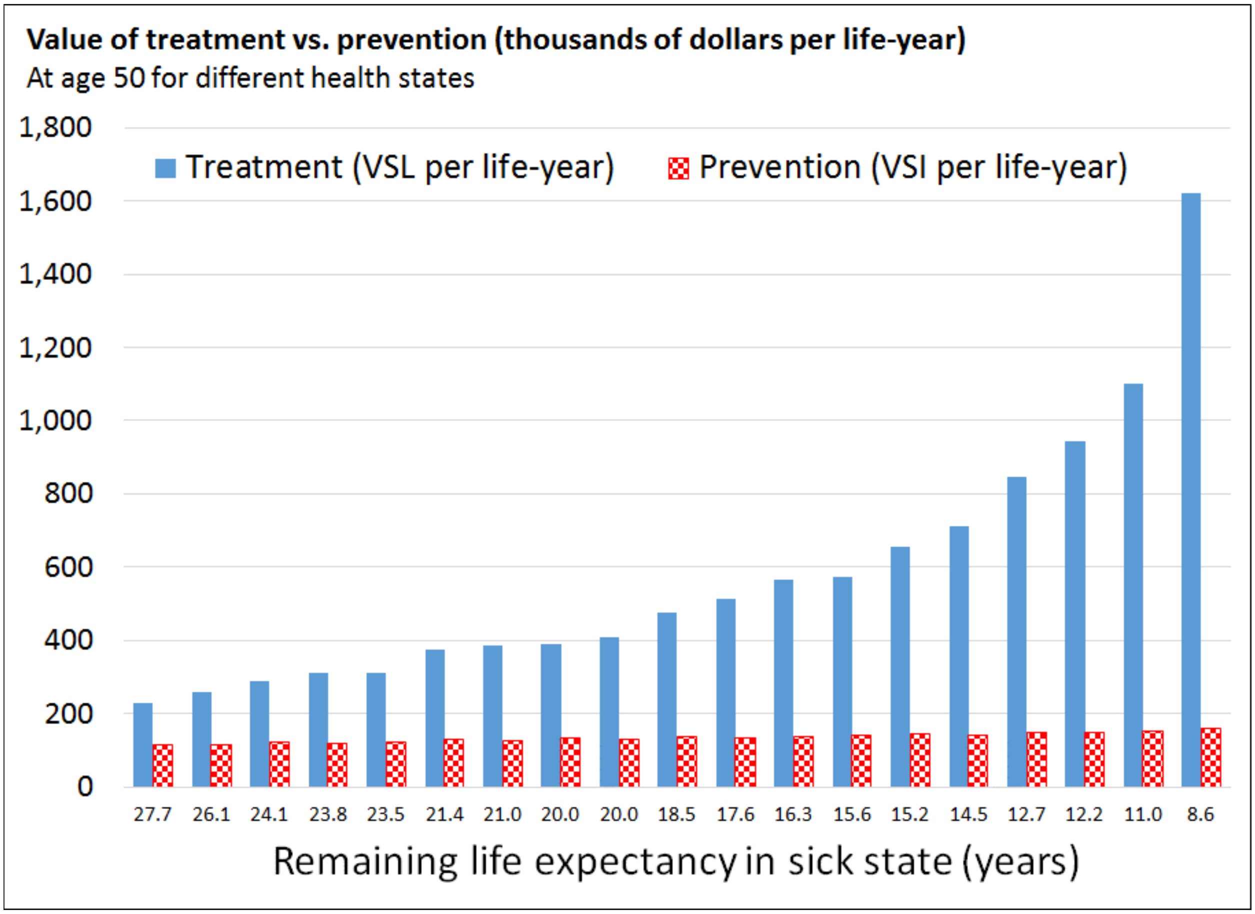
Notes: This figure plots an individual's consumption (left axis) and value of statistical life (right axis), as calculated by a life-cycle modeling exercise where mortality and quality of life are stochastic. The individual is healthy at age 50, but then falls ill twice, once at age 60 and then again at age 70. At age 60, the illness causes permanent difficulties with one routine activity of daily living (ADL). At age 70, she is diagnosed with two chronic conditions and subsequently has difficulties with two additional ADL's. In our data, this corresponds to transitioning from state 1 to state 6 at age 60, and then to state 18 at age 70. Summary statistics for these health states are available in Table 1.

Figure 4. The value of statistical life depends on an individual's health history



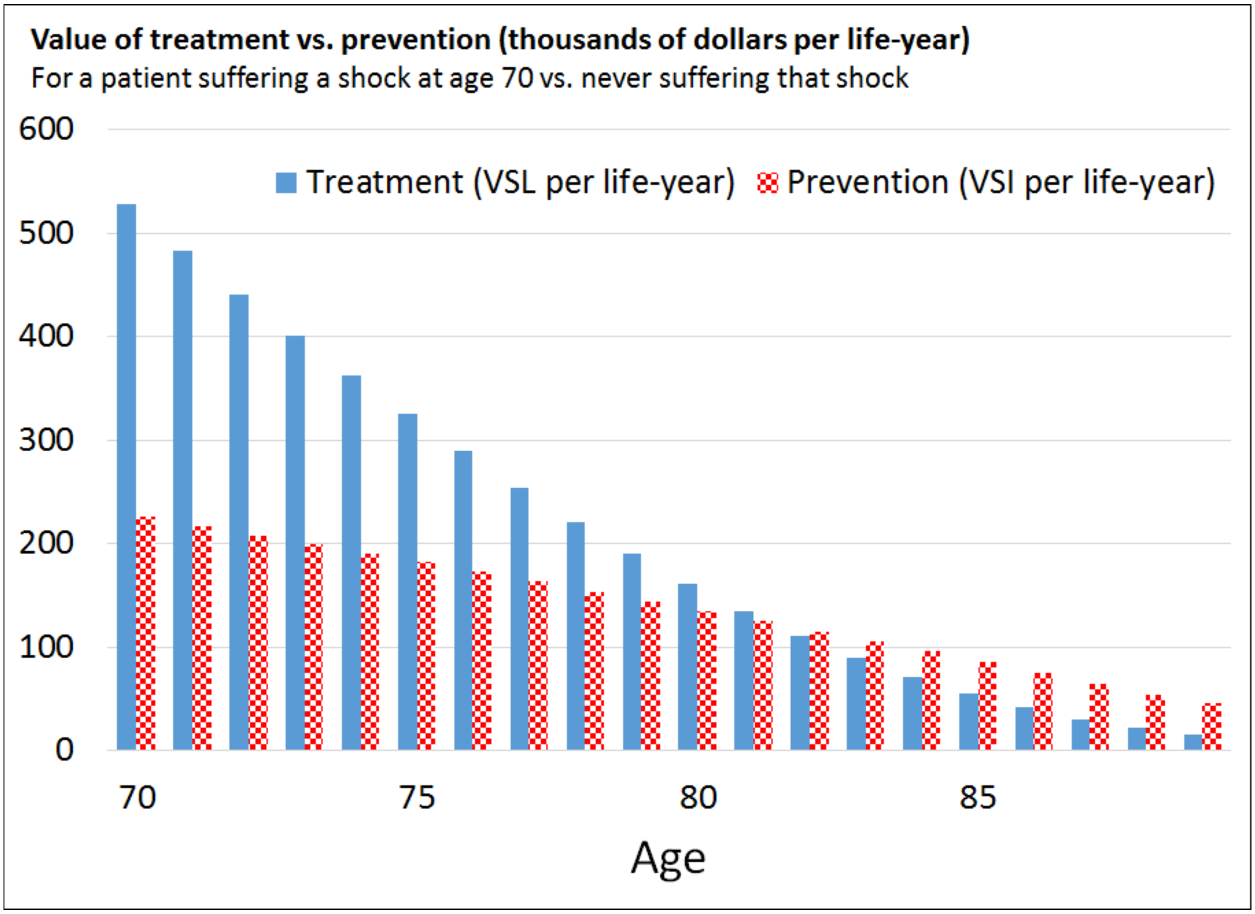
Notes: This figure reports the mean, 5th percentile, and 95th percentile of the value of statistical life (VSL) from a Monte Carlo simulation that is repeated 10,000 times. Each simulation begins at age 50 with a consumer in health state 1 ("healthy"). We then randomly generate a health state path $\{Y_{51}, Y_{52}, \ldots, Y_{100}\}$ and solve for optimal consumption and VSL using the methods described in Appendix C2. Differences in VSL at older ages are caused by differences in the evolution of people's health states.

Figure 5. Treatments for an ill patient are worth more than preventive care for a healthy individual



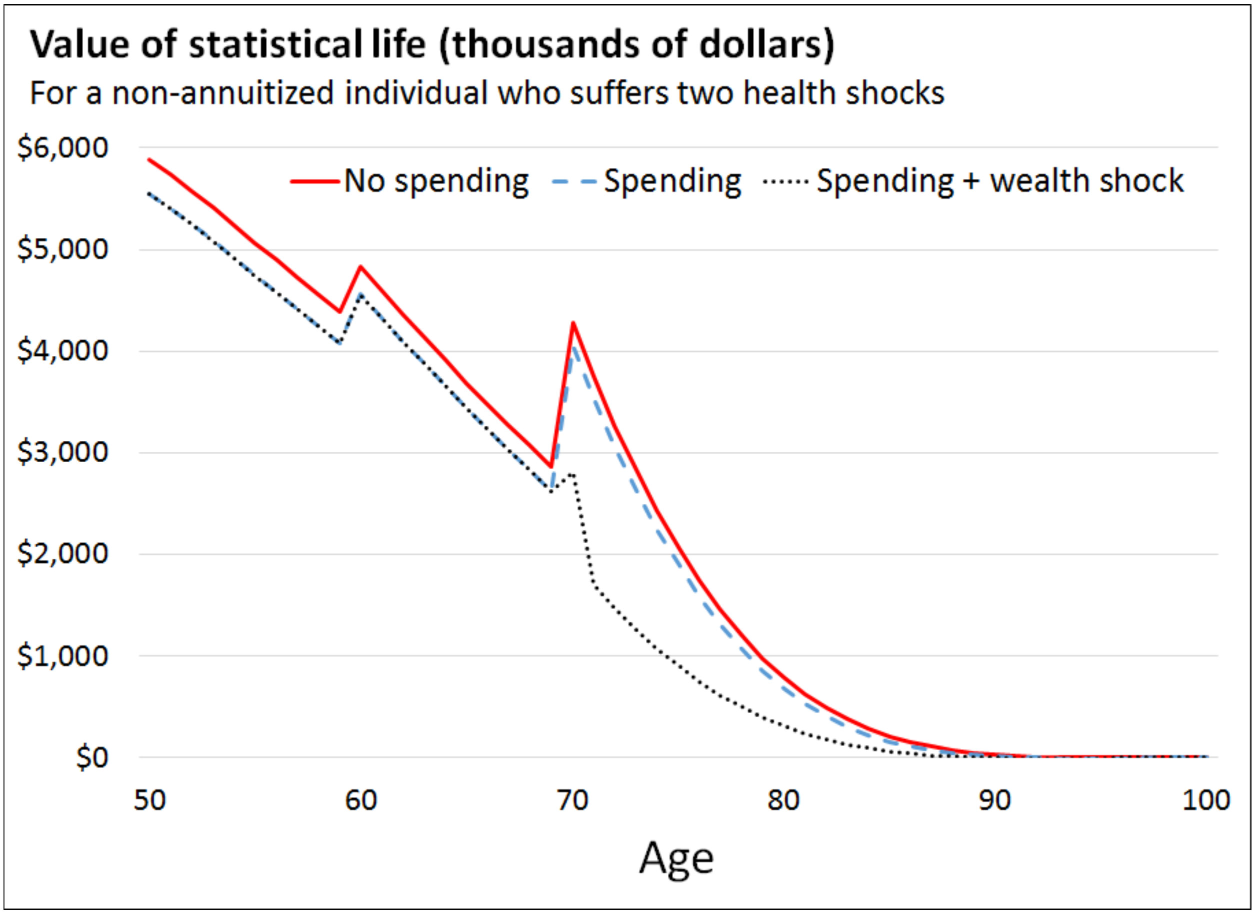
Notes: The solid blue bars report the value of statistical life (VSL) for an individual in one of 19 different sick states, divided by life expectancy in that state. The dotted red bars report the value of statistical illness (VSI) for a healthy individual (life expectancy: 30.4 years) divided by the reduction in life expectancy she would experience if she fell ill. The data plotted in this figure are also reported in columns (5) and (6) of Table 2.

Figure 6. The value of treatment relative to prevention declines with time since illness



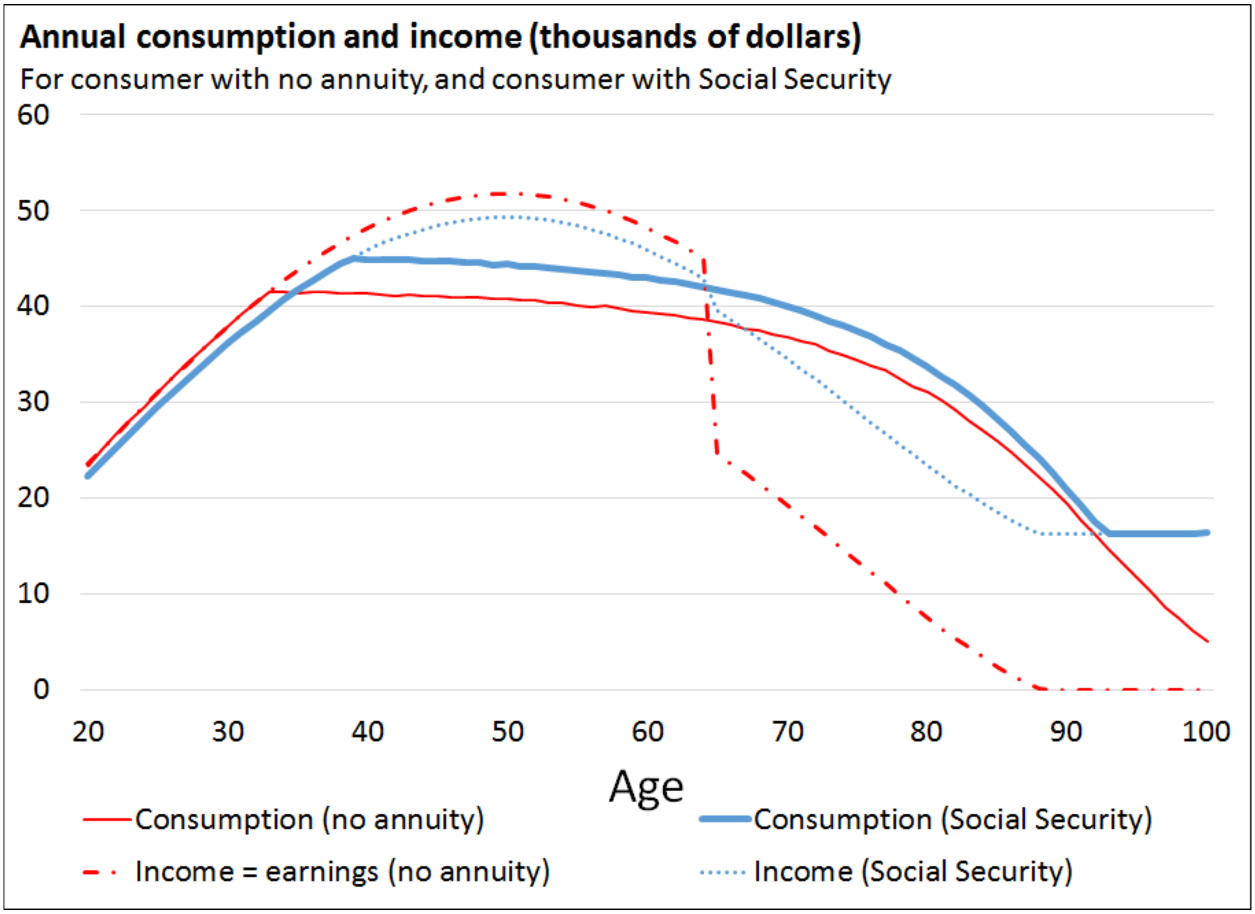
Notes: The solid blue bars report the value of statistical life (VSL) divided by life expectancy for the individual who suffered the health shock at age 70 depicted in Figure 3. The dotted red bars report the value of statistical illness (VSI) for a healthy individual divided by the *reduction* in life expectancy she would experience if she fell ill with the same disease.

Figure 7. Correlated spending shocks attenuate the rise in the value of statistical life following a health shock



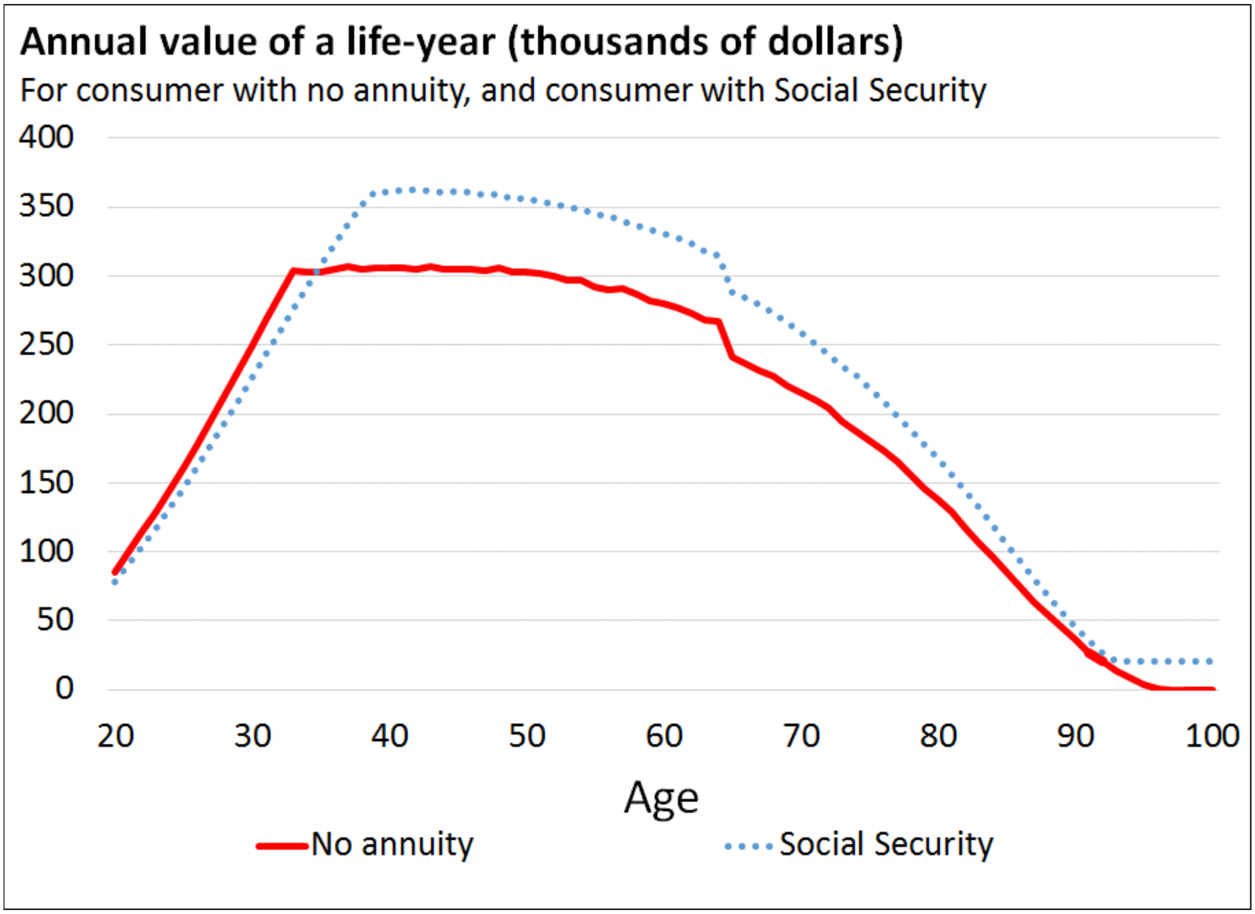
Notes: The solid red line, which reproduces the value of statistical life (VSL) estimates displayed in Figure 3, assumes that health shocks are not accompanied by medical spending shocks. The dashed blue line shows that VSL drops slightly following a health shock when we incorporate out-of-pocket medical spending shocks into the life-cycle model. The dotted black line additionally incorporates a wealth shock at age 70 that reduces the individual's wealth by 30 percent. Medical spending includes the expected effect of illness on both out-of-pocket healthcare costs and nursing home costs.

Figure 8. Life-cycle profiles of consumption and income when mortality is deterministic



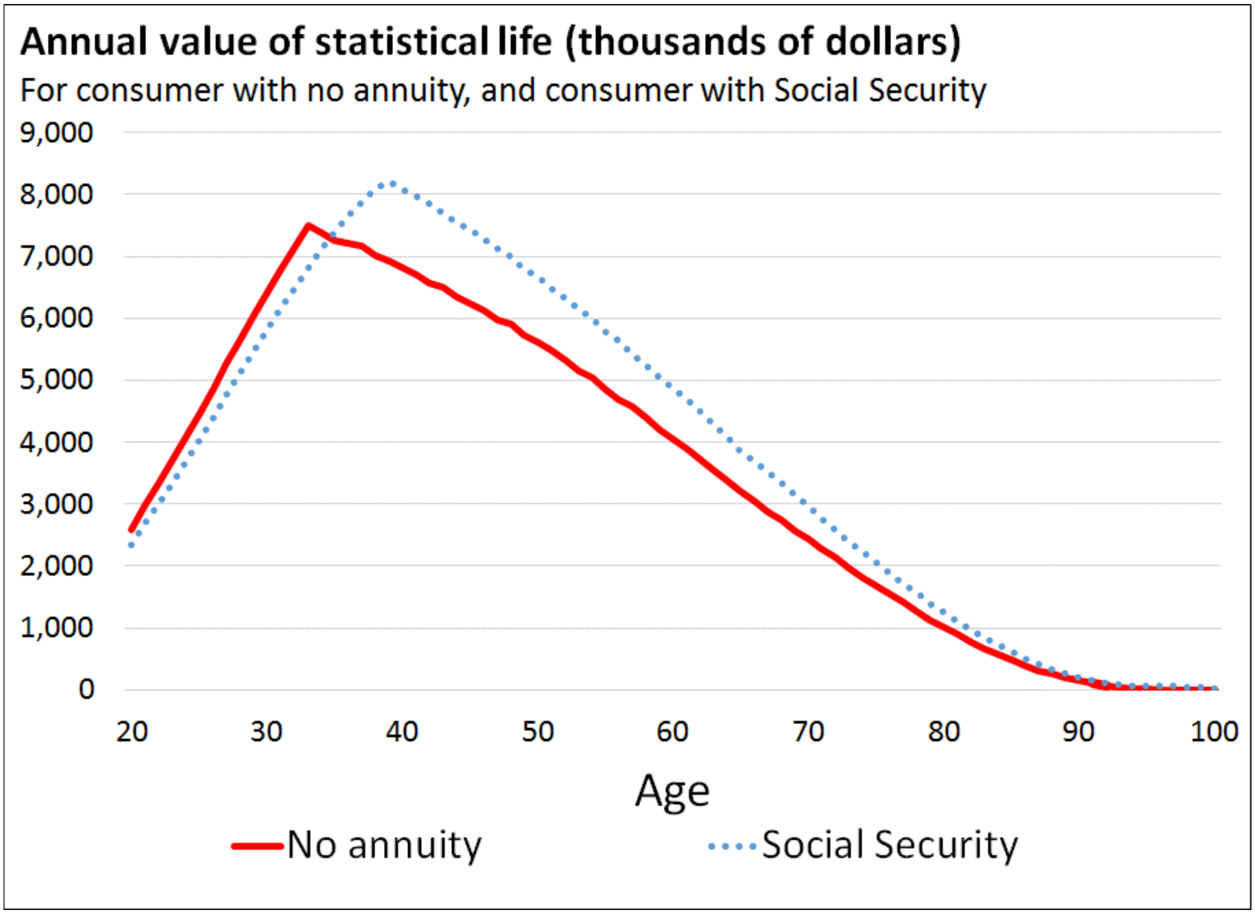
Notes: This figure plots consumption results from a life-cycle modeling exercise where mortality is deterministic. "Consumption (no annuity)" displays consumption for a consumer whose income equals her earnings. "Consumption (Social Security)" displays consumption for a consumer receiving typical Social Security benefits that are financed by an earnings tax. The net present value at age 20 of all future income is the same across both scenarios.

Figure 9. Life-cycle profile of the value of a life-year when mortality is deterministic



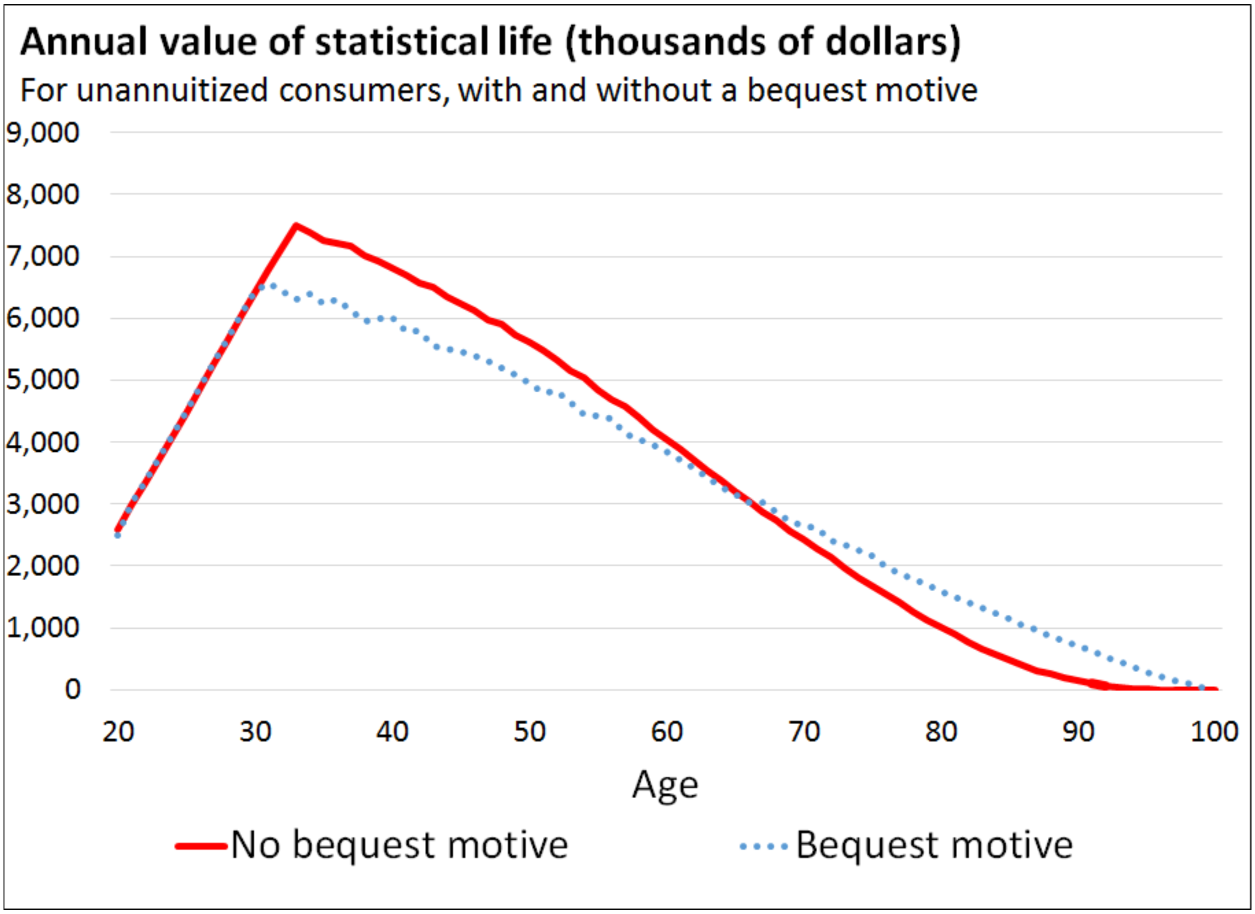
Notes: This figure plots the value of a life-year for the two scenarios displayed in Figure 8. The "No annuity" scenario assumes the consumer's income equals her earnings. The "Social Security" scenario assumes the consumer receives typical Social Security benefits that are financed by an earnings tax. The net present value at age 20 of all future income is identical in both scenarios.

Figure 10. Life-cycle profile of the value of statistical life when mortality is deterministic



Notes: This figure plots the value of statistical life for the two scenarios displayed in Figure 8. The "No annuity" scenario assumes the consumer's income equals her labor earnings. The "Social Security" scenario assumes the consumer receives typical Social Security benefits that are financed by an earnings tax. The net present value at age 20 of all future income is identical in both scenarios.

Figure 11. Similar to annuitization, a bequest motive shifts the value of statistical life towards older ages



Notes: This figure plots the value of statistical life in a setting with deterministic mortality and no annuity markets. The "No bequest motive" scenario is identical to the "No annuity" scenario depicted in Figure 10. The bequest motive specification is described at the end of Appendix C1.

APPENDIX (FOR ONLINE PUBLICATION ONLY)

The value of life depends greatly on the elasticity of intertemporal substitution, which under CRRA is equal to the inverse of γ, the coefficient of relative risk aversion. The specification in the main text, which sets $\gamma = 2$, calculated that Social Security raised the aggregate social value of post-1940 reductions by $11.5 trillion (10.5 percent). Appendix Table A1 and Appendix Table A2, which both replicate Table 4 from the main text, show that varying γ over the range $[1.5, 2.5]$ yields analogous increases that range from 8.3 percent to 14.2 percent.

The first two columns of Appendix Table A3 show that when a strong bequest motive is present, the increase in the aggregate social value of life attributable to Social Security is equal to $5.5 trillion (5.4 percent). Finally, the third column of Appendix Table A3 shows that fully annuitizing all wealth and future earnings at age 20 increases the aggregate value of life by $17.7 trillion (16 percent), relative to a setting with no annuity markets.

Appendix Figure A1 reports average out-of-pocket medical spending, by age, for a healthy individual in health state 1 and for a very sick individual in health state 20. These spending data include all inpatient, outpatient, prescription drug, and long-term care payments made by the individual, as estimated by the Future Elderly Model. The large increase in spending that occurs after age 80 is due primarily to the large costs of long-term care.

Appendix A provides proofs for lemmas and propositions stated in the main text. Appendix B provides descriptions of the data employed by the numerical models presented in Section IV, and Appendix C provides supporting calculations for those models. Finally, Appendix D provides derivations for the value of statistical life and the value of statistical illness for a fully annuitized consumer when mortality is stochastic.

Appendix Tables and Figures

Appendix Table A1. Aggregate social value of historical and prospective reductions in mortality when the degree of relative risk aversion is set equal to $\gamma = 2.5$ (billions of dollars)

	(1)	(2)	(3)
	No annuity	**Social Security**	**Social Security + 50%**
Historical reduction:			
1940-2010	\$222,046	\$253,546	\$269,951
1970-2010	\$109,580	\$126,291	\$135,146
10% reduction, all ages:			
All causes	\$23,879	\$27,569	\$29,566
Cancer	\$6,943	\$8,081	\$8,697
Diabetes	\$762	\$885	\$951
Heart disease	\$5,068	\$5,910	\$6,374
Homicide	\$189	\$187	\$184
Infectious diseases	\$349	\$408	\$441

Notes: These aggregate values were calculated using the 2015 US population by age. Panel A reports the current value of historical reductions in all-cause mortality. Panel B reports the value of a 10 percent prospective reduction in mortality. Column (1) presents estimates under the assumption that individuals have no annuities in retirement. Column (2) presents estimates under the assumption that individuals receive typical Social Security benefits that are financed by an earnings tax. Column (3) increases the generosity of Social Security by 50%, financed by an increase in the earnings tax. The net present value of individuals' wealth at age 20 is the same across all three columns. The degree of relative risk aversion, γ, is equal to the inverse of the elasticity of intertemporal substitution. In the main text, we assume that $\gamma = 2$.

Appendix Table A2. Aggregate social value of historical and prospective reductions in mortality when the degree of relative risk aversion is set equal to $\gamma = 1.5$ (billions of dollars)

	(1)	(2)	(3)
	No annuity	**Social Security**	**Social Security + 50%**
Historical reduction:			
1940-2010	\$27,121	\$29,381	\$30,465
1970-2010	\$5,750	\$6,277	\$6,555
10% reduction, all ages:			
All causes	\$1,661	\$1,822	\$1,903
Cancer	\$183	\$200	\$209
Diabetes	\$1,185	\$1,310	\$1,379
Heart disease	\$63	\$61	\$59
Homicide	\$80	\$89	\$94
Infectious diseases	\$0	\$0	\$0

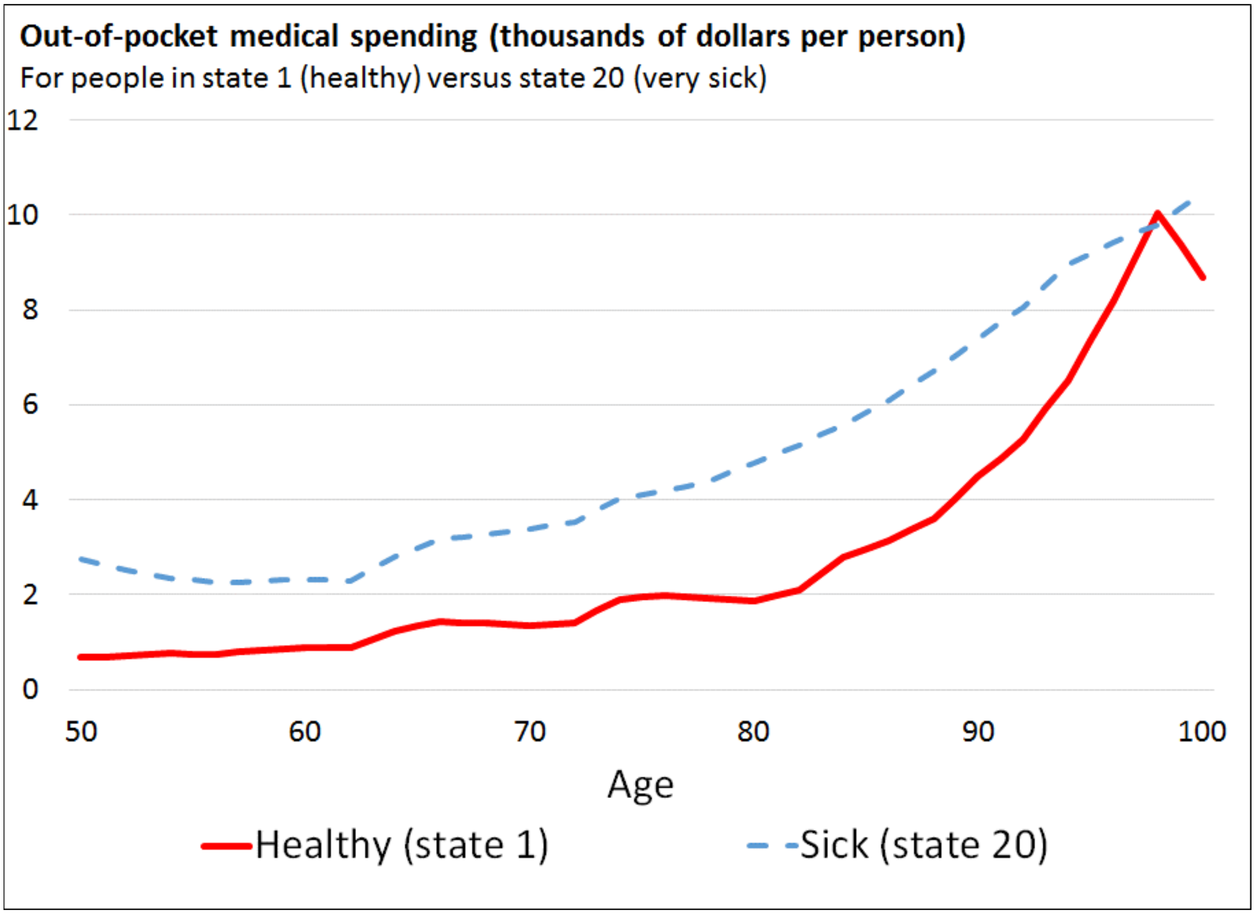
Notes: These aggregate values were calculated using the 2015 US population by age. Panel A reports the current value of historical reductions in all-cause mortality. Panel B reports the value of a 10 percent prospective reduction in mortality. Column (1) presents estimates under the assumption that individuals have no annuities in retirement. Column (2) presents estimates under the assumption that individuals receive typical Social Security benefits that are financed by an earnings tax. Column (3) increases the generosity of Social Security by 50%, financed by an increase in the earnings tax. The net present value of individuals' wealth at age 20 is the same across all three columns. The degree of relative risk aversion, γ, is equal to the inverse of the elasticity of intertemporal substitution. In the main text, we assume that $\gamma = 2$.

Appendix Table A3. Aggregate social value of historical and prospective reductions in mortality when a bequest motive is present or when consumer is fully annuitized (billions of dollars)

	(1)	(2)	(3)	(4)
	Bequest motive		**No bequest motive**	
	No annuity	**Social Security**	**No annuity**	**Full annuitization**
Historical reduction:				
1940-2010	$102,744	$108,261	$109,356	$127,030
1970-2010	$50,110	$53,081	$53,492	$60,571
10% reduction, all ages:				
All causes	$11,042	$11,758	$11,550	$13,403
Cancer	$3,150	$3,362	$3,348	$3,708
Diabetes	$348	$371	$368	$412
Heart disease	$2,338	$2,512	$2,425	$2,755
Homicide	$99	$95	$105	$173
Infectious diseases	$163	$176	$166	$192

Notes: The bequest motive specification is described at the end of Appendix C1. These aggregate values were calculated using the 2015 US population by age. Panel A reports the current value of historical reductions in all-cause mortality. Panel B reports the value of a 10 percent prospective reduction in mortality. Column (1) presents estimates under the assumption that individuals have no annuities. Column (2) presents estimates under the assumption that individuals receive typical Social Security benefits that are financed by an earnings tax. Column (3) increases the generosity of Social Security by 50%, financed by an increase in the earnings tax. The net present value of individuals' wealth at age 20 is the same across all three columns.

Appendix Figure A1. Annual out-of-pocket medical spending for a healthy person versus a very sick patient



Notes: These medical spending estimates include out-of-pocket spending on both health care and nursing homes. State 1 corresponds to a healthy individual with no impaired activities of daily living (ADL) and no chronic conditions. State 20 corresponds to an individual with three or more ADL's and four or more chronic conditions. Additional characteristics for these health states are provided in Table 1. These estimates are provided by the Future Elderly Model, which is described in greater detail in Appendix B2.

A. Mathematical proofs of results from main text

Proof of Lemma 1:

Recall that the transition intensities $\lambda_{ij}(t) = 0\ \forall j < i$. The optimization problem in the absorbing state n is therefore a standard deterministic problem. We can contemplate a successive solution strategy by starting in state n and then moving sequentially to state $n-1$, $n-2$, etc. Thus, we can consider the deterministic optimization problem for an arbitrary state i by taking $V(t,w,j), j > i$, as given (exogenous):

$$V(0,W_0,i) = \max_{c_i(t)} \left\{ \int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right\}$$

subject to:

$$\frac{\partial W_i(t)}{\partial t} = rW_i(t) + m_i(t) - c_i(t), W_i(0) = W_0$$

Optimal consumption and wealth in state i are denoted by $c_i(t)$ and $W_i(t)$, respectively. Denote the optimal value-to-go function as:

$$\tilde{V}(u, W_i(u), i) = \max_{c_i(t)} \left\{ \int_u^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \right\}$$

Setting $\tilde{V}(t, W_i(t), i) = e^{-\rho t} \tilde{S}(i,t) V(t, W_i(t), i)$ then demonstrates that $V(\cdot)$ satisfies the HJB (11) for i. See Theorem 1 and the proof of Theorem 2 in Parpas and Webster (2013) for additional details and intuition behind this result.

QED

Proof of Lemma 2:

From (12), the marginal utility of life-extension is:

$$\begin{aligned}
\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon} \int_0^T e^{-\rho t} \exp\left\{ -\int_0^t \big(\mu(s) - \varepsilon \delta(s)\big) + \sum_{j>i} \lambda_{ij}(s)\, ds \right\} \left(u\big(c_i^{\varepsilon}(t), q_i(t)\big) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i^{\varepsilon}(t), j) \right) dt \Bigg|_{\varepsilon=0} \\
&= \int_0^T e^{-\rho t} \left(\int_0^t \delta(s) ds \right) \tilde{S}(i,t) \left(u\big(c_i(t), q_i(t)\big) + \sum_{j>i} \lambda_{ij}(t) V(t, W_i(t), j) \right) dt \\
&\quad + \int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u_c\big(c_i(t), q_i(t)\big) \frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon} + \sum_{j>i} \lambda_{ij}(t) \frac{\partial V(t, W_i(t), j)}{\partial W_i(t)} \frac{\partial W_i^{\varepsilon}(t)}{\partial \varepsilon} \right) dt \Bigg|_{\varepsilon=0}
\end{aligned}$$

where $c_i^{\varepsilon}(t)$ and $W_i^{\varepsilon}(t)$ represent the equilibrium variations in $c_i(t)$ and $W_i(t)$ caused by this perturbation. We conclude the proof by showing that the second term in the last equality is equal to 0. Note that along this path, wealth at time t is equal to:

$$W_i(t) = W_0 e^{rt} + \int_0^t e^{r(t-s)} m_i(s) ds - \int_0^t e^{r(t-s)} c_i(s) ds,$$

which implies $\frac{\partial W_i^{\varepsilon}(t)}{\partial \varepsilon} = -\int_0^t e^{r(t-s)} \frac{\partial c_i^{\varepsilon}(s)}{\partial \varepsilon} ds$. From the solution to the costate equation, we know that:

$$e^{-\rho t}\tilde{S}(i,t)u_c\big(c_i(t),q_i(t)\big) = \left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}ds\right]e^{-rt} + \theta^{(i)}e^{-rt}$$

Thus, we can rewrite the second term in the expression for $\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0}$ above as:

$$\int_0^T\left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}ds + \theta^{(i)}\right]e^{-rt}\frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon}dt$$
$$\left. - \int_0^T e^{-\rho t}\tilde{S}(i,t)\sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t,W_i(t),j)}{\partial W_i(t)}\int_0^t e^{r(t-s)}\frac{\partial c_i^\varepsilon(s)}{\partial \varepsilon}ds\,dt\right|_{\varepsilon=0}$$
$$= \int_0^T\left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}ds\right]e^{-rt}\frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon}dt$$
$$\left. - \int_0^T\left[\int_t^T e^{(r-\rho)s}\tilde{S}(i,s)\sum_{j>i}\lambda_{ij}(s)\frac{\partial V(s,W_i(s),j)}{\partial W_i(s)}ds\right]e^{-rt}\frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon}dt + \int_0^T \theta^{(i)}e^{-rt}\frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon}dt\right|_{\varepsilon=0}$$
$$= \theta^{(i)}\frac{\partial}{\partial \varepsilon}\left.\underbrace{\int_0^T e^{-rt}c_i^\varepsilon(t)dt}_{W_0+\int_0^T e^{-rt}m_i(t)dt}\right|_{\varepsilon=0}$$
$$= 0$$

where, as in the deterministic case, the last equality follows from application of the budget constraint.

QED

Proof of Lemma 3:

The proof proceeds by induction on $i \leq n$. For the base case $i = n$, in which no state transitions are possible, the solution to the costate equation (13) simplifies to:[29]

$$\begin{aligned}
p_\tau^{(n)} = \theta^{(n)}e^{-r\tau} &= \exp\left\{-\int_0^\tau \rho + \overline{\mu}_n(s)ds\right\}u_c(c_n(\tau),q_n(\tau)) \\
&= \theta^{(n)}e^{-rt}e^{-r(\tau-t)} \\
&= p_t^{(n)}e^{-r(\tau-t)} \\
&= \exp\left\{-\int_0^t \rho + \overline{\mu}_n(s)ds\right\}u_c(c_n(t),q_n(t))e^{-r(\tau-t)}
\end{aligned}$$

This then implies:

$$u_c(c_n(t),q_n(t)) = e^{r(\tau-t)}e^{-\rho(\tau-t)}\exp\left\{-\int_t^\tau \overline{\mu}_n(s)ds\right\}u_c(c_n(\tau),q_n(\tau))$$

which shows that the lemma holds for $i = n$.

[29] When no transitions are possible, the solution reduces to the first-order condition presented in Section II.B.

For the induction step, suppose the lemma is true for $j > i$, $1 \leq i \leq n-1$. For any subinterval $[0, \tau]$, the solution of the costate equation can be written as:

$$p_t^{(i)} = \left[\int_t^\tau e^{(r-\rho)s} \exp\left\{ -\int_0^s \overline{\mu}_i(u) + \sum_{j>i} \lambda_{ij}(u)\, du \right\} \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} + \theta(\tau, i) e^{-rt} \qquad \text{(A1)}$$

where $\theta(\tau, i)$ is a constant that depends on the choice of τ and i. (Take the derivative of $p_t^{(i)}$ with respect to t to verify.) Evaluating equation (A1) at $t = \tau$ and combining with equation (14) from the main text yields:

$$p_\tau^{(i)} = \theta(\tau, i) e^{-r\tau} = \exp\left\{ -\int_0^\tau \rho + \overline{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s)\, ds \right\} u_c\big(c_i(\tau), q_i(\tau)\big)$$

which implies:

$$\theta(\tau, i) = e^{(r-\rho)\tau} \exp\left\{ -\int_0^\tau \overline{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s)\, ds \right\} u_c\big(c_i(\tau), q_i(\tau)\big) \qquad \text{(A2)}$$

Plugging equations (14) and (A2) into equation (A1) yields:

$$\begin{aligned} &u_c\big(c_i(t), q_i(t)\big) \exp\left\{ -\int_0^t \rho + \overline{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s)\, ds \right\} \\ &\quad = \left[\int_t^\tau e^{(r-\rho)s} \exp\left\{ -\int_0^s \overline{\mu}_i(u) + \sum_{j>i} \lambda_{ij}(u)\, du \right\} \sum_{j>i} \lambda_{ij}(s) \frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} ds \right] e^{-rt} \\ &\qquad + e^{-rt} e^{(r-\rho)\tau} \exp\left\{ -\int_0^\tau \overline{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s)\, ds \right\} u_c\big(c_i(\tau), q_i(\tau)\big) \end{aligned}$$

Since $\frac{\partial V(s, W_i(s), j)}{\partial W_i(s)} = u_c\big(c(s, W_i(s), j), q_j(s)\big)$ from the first-order condition in the HJB for state j, we obtain:

$$\begin{aligned} u_c\big(c_i(t), q_i(t)\big) &= \int_t^\tau e^{(r-\rho)(s-t)} \exp\left\{ -\int_t^s \overline{\mu}_i(u) + \sum_{j>i} \lambda_{ij}(u)\, du \right\} \sum_{j>i} \lambda_{ij}(s) u_c\big(c(s, W_i(s), j), q_j(s)\big)\, ds \\ &\quad + e^{(r-\rho)(\tau-t)} \exp\left\{ -\int_t^\tau \overline{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s)\, ds \right\} u_c\big(c_i(\tau), q_i(\tau)\big) \\ &= \int_t^\tau e^{(r-\rho)(s-t)} \exp\left\{ -\int_t^s \overline{\mu}_i(u) + \sum_{j>i} \lambda_{ij}(u)\, du \right\} \sum_{j>i} \lambda_{ij}(s) \mathbb{E}\left[e^{(r-\rho)(\tau-s)} \exp\left\{ -\int_s^\tau \mu(s) ds \right\} u_c\big(c(\tau, W(\tau), Y_\tau), q_{Y_\tau}(\tau)\big) \middle| Y_s = j, W(s) \right. \\ &\quad \left. = W_i(s) \right] ds + e^{(r-\rho)(\tau-t)} \exp\left\{ -\int_t^\tau \overline{\mu}_i(s) + \sum_{j>i} \lambda_{ij}(s)\, ds \right\} u_c\big(c_i(\tau), q_i(\tau)\big) \\ &= \mathbb{E}\left[e^{(r-\rho)(\tau-s)} \exp\left\{ -\int_t^\tau \mu(s) ds \right\} u_c\big(c(\tau, W(\tau), Y_\tau), q_{y_\tau}(\tau)\big) \middle| Y_t = i, W(t) = W_i(t) \right] \end{aligned}$$

where the second equality follows from the induction hypothesis.

QED

Proof of Proposition 4:

Choosing once again the Dirac delta function for $\delta(\cdot)$ in **Lemma 2** yields:

$$\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T \left[e^{-\rho t} \tilde{S}(i,t) \left(u\big(c_i(t), q_i(t)\big) + \sum_{j>i} \lambda_{ij}(t) \, V(t, W_i(t), j) \right) \right] dt$$
$$= \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) u\big(c(t), q_{Y_t}(t)\big) dt \,\middle|\, Y_0 = i, W(0) = W_0 \right]$$

Dividing the result by the marginal utility of wealth at time $t = 0$ then yields the value of statistical life given by equation (15):

$$VSL(i) = \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) \frac{u\left(c(t), q_{Y_t}(t)\right)}{u\left(c(0), q_{Y_0}(0)\right)} dt \,\middle|\, Y_0 = i, W(0) = W_0 \right]$$

Applying **Lemma 3** for $t = 0$ allows us to rewrite VSL as:

$$VSL(i) = \mathbb{E}\left[\int_0^T e^{-\rho t} \frac{S(t) u\big(c(t), q_{Y_t}(t)\big)}{\mathbb{E}\left[e^{(r-\rho)t} \exp\left\{ -\int_0^t \mu(s) ds \right\} u_c\big(c(t), q_{Y_t}(t)\big) \,\middle|\, Y_0 = i, W(0) = W_0 \right]} dt \,\middle|\, Y_0 = i, W(0) = W_0 \right]$$
$$= \mathbb{E}\left[\int_0^T e^{-rt} \frac{S(t) u\big(c(t), q_{Y_t}(t)\big)}{\mathbb{E}\left[\exp\left\{ -\int_0^t \mu(s) ds \right\} u_c\big(c(t), q_{Y_t}(t)\big) \,\middle|\, Y_0 = i, W(0) = W_0 \right]} dt \,\middle|\, Y_0 = i, W(0) = W_0 \right]$$

Exchanging expectation and integration then yields:

$$VSL(i) = \int_0^T e^{-rt} v(i,t) dt$$

where the value of a life-year, $v(i,t)$, is equal to the expected utility of consumption normalized by the expected marginal utility of consumption:

$$v(i,t) = \frac{\mathbb{E}\left[S(t) u\left(c(t), q_{Y_t}(t)\right) \,\middle|\, Y_0 = i, W(0) = W_0 \right]}{\mathbb{E}\left[S(t) u_c\left(c(t), q_{Y_t}(t)\right) \,\middle|\, Y_0 = i, W(0) = W_0 \right]}$$

QED

Proof of Proposition 5:

Without loss of generality, we will prove the proposition for the case where the consumer transitions from state 1 to state 2 at time $t = 0$. Because we hold quality of life constant, we omit $q_i(t)$ in the notation below in order to keep the presentation concise.

We want to prove that $c_2(0) \geq c_1(0)$. Assume by way of contradiction that $c_2(0) < c_1(0)$. We will show that this implies $c_2(t) < c_1(t)$ for all $t > 0$, which is a contradiction since the feasible consumption plan $c_1(\cdot)$ dominates $c_2(\cdot)$.

We proceed by inductively constructing a sequence $0 < t_1 < t_2$ ... where for each element in the sequence:

$$c_2(t_i) < c_1(t_i)$$
$$W_1(t_i) \leq W_2(t_i)$$

$$p_{t_i}^{(1)} < \exp\left\{-\int_0^{t_i} \lambda_{12}(s)ds\right\} p_{t_i}^{(2)}$$

To construct the sequence, for the base case $i = 1$, we first note that from the first-order condition (14), we obtain:

$$p_0^{(1)} = u_c\big(c_1(0)\big) < u_c\big(c_2(0)\big) = p_0^{(2)}$$

The costate equation (13) then implies:

$$\begin{aligned}\dot{p}_0^{(1)} &= -p_0^{(1)} r - \lambda_{12}(0) u_c\big(c_2(0)\big) \\ &= -p_0^{(1)} \left[r + \lambda_{12}(0) \underbrace{\frac{u_c\big(c_2(0)\big)}{u_c\big(c_1(0)\big)}}_{>1} \right] \\ &< -p_0^{(1)} [r + \lambda_{12}(0)] = \left.\frac{\partial g(t)}{\partial t}\right|_{t=0}\end{aligned}$$

where $g(t) = p_0^{(1)} \exp\left\{-\int_0^t r + \lambda_{12}(s)ds\right\}$. Hence, there exists a $t_1 > t_0 = 0$ such that:

$$p_t^{(1)} \le g(t) < p_0^{(2)} \exp\left\{-\int_0^t \big(r + \lambda_{12}(s)\big)ds\right\} = p_t^{(2)} \exp\left\{-\int_0^t \lambda_{12}(s)ds\right\}, 0 \le t \le t_1$$

which together with the first-order condition (14) implies:

$$e^{-\rho t} \exp\left\{-\int_0^t \left(\overline{\mu}_1(s) + \lambda_{12}(s)\right) ds\right\} u_c\big(c_1(t)\big) < e^{-\rho t} \exp\left\{-\int_0^t \left(\overline{\mu}_2(s) + \lambda_{12}(s)\right) ds\right\} u_c\big(c_2(t)\big), 0 \le t \le t_1$$

so that $c_1(t) > c_2(t), 0 \le t \le t_1$. Since $m_1(s) \le m_2(s)\ \forall s$, this in turn implies $W_1(t_1) \le W_2(t_1)$.

For the induction step, suppose that the following properties also hold for $i \ge 1$:

$$\begin{aligned} c_2(t_i) &< c_1(t_i) \\ W_1(t_i) &\le W_2(t_i) \\ p_{t_i}^{(1)} &< \exp\left\{-\int_0^{t_i} \lambda_{12}(s)ds\right\} p_{t_i}^{(2)} \end{aligned}$$

The induction hypothesis implies:

$$c(t_i, W_1(t_i), 2) \le c(t_i, W_2(t_i), 2) = c_2(t_i) < c_1(t_i)$$

so that:

$$\begin{aligned}\dot{p}_{t_i}^{(1)} &= -p_{t_i}^{(1)} r - e^{-\rho t_i} \tilde{S}(1, t_i) \lambda_{12}(t_i) u\big(c(t_i, W_1(t_i), 2)\big) \\ &= -p_{t_i}^{(1)} \left[r + \lambda_{12}(t_i) \underbrace{\frac{u_c\big(c(t_i, W_1(t_i), 2)\big)}{u_c\big(c_1(t_i)\big)}}_{>1} \right] \\ &< -p_{t_i}^{(1)} [r + \lambda_{12}(t_i)] = \left.\frac{\partial \tilde{g}(t_i)}{\partial t}\right|_{t_i=0}\end{aligned}$$

with $\tilde{g}(t_i) = p_{t_i}^{(1)} \exp\left\{-\int_{t_i}^{t}\left(r + \lambda_{12}(s)\right)ds\right\}$. Hence, there exists a $t_{i+1} > t_i$ such that:

$$p_t^{(1)} \leq \tilde{g}(t)$$
$$< \exp\left\{-\int_0^{t_i} \lambda_{12}(s)ds\right\} p_{t_i}^{(2)} \exp\left\{-\int_{t_i}^{t}\left(r + \lambda_{12}(s)\right)ds\right\} = p_t^{(2)} \exp\left\{-\int_0^{t} \lambda_{12}(s)ds\right\}, t_i \leq t \leq t_{i+1}$$

In particular, again with the first-order condition (14) for all $t_i \leq t \leq t_{i+1}$:

$$\exp\left\{-\int_0^t \left(\overline{\mu}_1(s) + \lambda_{12}(s)\right) ds\right\} u_c\left(c_1(t)\right) < \exp\left\{-\int_0^t \left(\overline{\mu}_2(s) + \lambda_{12}(s)\right) ds\right\} u_c\left(c_2(t)\right)$$

which in turn implies $u_c\left(c_1(t)\right) < u_c\left(c_2(t)\right)$ and $c_2(t) < c_1(t)$. Once again, together with the assumption $m_1(s) \leq m_2(s)$, this implies $W_1(t_{i+1}) \leq W_2(t_{i+1})$.

Thus, we have proven the existence of the sequence. We then obtain $c_2(t) < c_1(t)$ $\forall t$ by noting that $\{t_i\}_{i\geq 0}$ strictly increases due to the uniformly boundedness condition on $\lambda_{12}(t)$, which is the desired contradiction.

We note that this proof implies that the consumption paths $c_1(t)$ and $c_2(t)$ cross (at most) once. As soon as $c_1(t)$ exceeds $c_2(t)$ for some time t_0, $c_1(t)$ will exceed $c_2(t)$ for $t > t_0$. However, we have that $c_2(t)$ exceeds $c_1(t)$ prior to t_0. In particular, consumption jumps up at the transition point. See Figure 2 for an illustration.

QED

Proof of Proposition 6:

Without loss of generality, consider the case $t = 0$, as depicted in Figure 2. From **Proposition 5** it is clear that $c_1(t)$ and $c_2(t)$ are decreasing, $c_2(0) \geq c_1(0)$, $c_2(t) \geq c_1(t)$ for $t \leq t_0$, and $c_2(t) \leq c_1(t)$ for $t > t_0$. Making use of the assumption that no state transitions occur for $t > 0$, we have that:

$$VSL(2,0) = \int_0^T e^{-rt} \frac{S_2(t)u\left(c_2(t)\right)}{S_2(t)u_c\left(c_2(t)\right)} dt$$
$$= \int_0^T e^{-rt} \frac{u\left(c_2(t)\right)}{u_c\left(c_2(t)\right)} dt$$

and:

$$VSL(1,0) = \int_0^T e^{-rt} \frac{u\left(c_1(t)\right)}{u_c\left(c_1(t)\right)} dt$$

Let $Y(x) = \frac{u(x)}{u_c(x)}$. Under the stated assumptions on preferences, we have that:[30]

$$Y'(x) = 1 - \frac{u(x)u_{cc}(x)}{\left(u_c(x)\right)^2} > 0,$$

[30] Strictly speaking, this proof requires only that $Y'(x) > 0$ and $Y''(x) > 0$. The stated assumptions on preferences are therefore sufficient, but not necessary.

$$Y''(x) = \frac{2\big(u_{cc}(x)\big)^2 u(x) - u_c^2(x)u_{cc}(x) - u_c(x)u(x)u_{ccc}(x)}{\big(u_c(x)\big)^3} > 0$$

Employing Taylor's theorem then implies that for some $\xi(t)$ that lies in-between $c_1(t)$ and $c_2(t)$:

$$\begin{aligned}
VSL(2,0) &= \int_0^T e^{-rt} Y\big(c_2(t)\big)\,dt \\
&= \int_0^T e^{-rt}\left[Y\big(c_1(t)\big) + [c_2(t) - c_1(t)]Y'\big(c_1(t)\big) + \underbrace{\frac{1}{2}[c_2(t) - c_1(t)]^2 Y''\big(\xi(t)\big)}_{>0}\right]dt \\
&\geq \int_0^T e^{-rt}\,Y\big(c_1(t)\big)dt + \int_0^{t_0} e^{-rt}\,Y'\big(c_1(t)\big)\underbrace{[c_2(t) - c_1(t)]}_{\geq 0}dt + \int_{t_0}^T e^{-rt}\,Y'\big(c_1(t)\big)\underbrace{[c_2(t) - c_1(t)]}_{\leq 0}dt \\
&\geq \int_0^T e^{-rt}\,Y\big(c_1(t)\big)dt + \int_0^{t_0} e^{-rt}\,Y'\big(c_1(t_0)\big)[c_2(t) - c_1(t)]dt + \int_0^{t_0} e^{-rt}\,Y'\big(c_1(t_0)\big)[c_2(t) - c_1(t)]dt \\
&= \int_0^T e^{-rt}\,Y\big(c_1(t)\big)dt + Y'\big(c_1(t_0)\big)\underbrace{\left[\int_0^T e^{-rt}\,c_2(t)dt - \int_0^T e^{-rt}\,c_1(t)dt\right]}_{=0} \\
&= \int_0^T e^{-rt}\,Y\big(c_1(t)\big)dt \\
&= VSL(1,0)
\end{aligned}$$

where the final step follows from the budget constraint.

QED

Proof of Proposition 7:

From (12), the marginal utility of preventing an illness or death is:

$$\begin{aligned}
\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} &= \frac{\partial}{\partial \varepsilon}\int_0^T e^{-\rho t}\exp\left\{-\int_0^t \big(\bar{\mu}_i(s) - \varepsilon\delta_{i,N+1}(t)\big) + \sum_{j>i}\big(\lambda_{ij}(s) - \varepsilon\delta_{ij}(s)\big)\,ds\right\}\Bigg(u\big(c_i^\varepsilon(t), q_i(t)\big) \\
&\qquad \left.+ \sum_{j>i}\big(\lambda_{ij}(t) - \varepsilon\delta_{ij}(t)\big)V(t, W_i^\varepsilon(t), j)\Bigg)dt\right|_{\varepsilon=0} \\
&= \int_0^T e^{-\rho t}\tilde{S}(i,t)\left[\left(\int_0^t \sum_{j>i}\delta_{ij}(s)\,ds\right)\left(u(c_i(t), q_i(t)) + \sum_{j>i}\lambda_{ij}(t)\,V(t, W_i(t), j)\right) - \sum_{j>i}\delta_{ij}(t)\,V(t, W_i(t), j)\right]dt \\
&\qquad + \int_0^T e^{-\rho t}\tilde{S}(i,t)\left(u_c\big(c_i^\varepsilon(t), q_i(t)\big)\frac{\partial c_i^\varepsilon(t)}{\partial \varepsilon} + \sum_{j>i}\lambda_{ij}(t)\frac{\partial V(t, W_i(t), j)}{\partial W}\frac{\partial W_i^\varepsilon(t)}{\partial \varepsilon}\right)dt
\end{aligned}$$

Following the same argument as in the VSL case, the second term in the last equality is equal to 0.

QED

B. Data

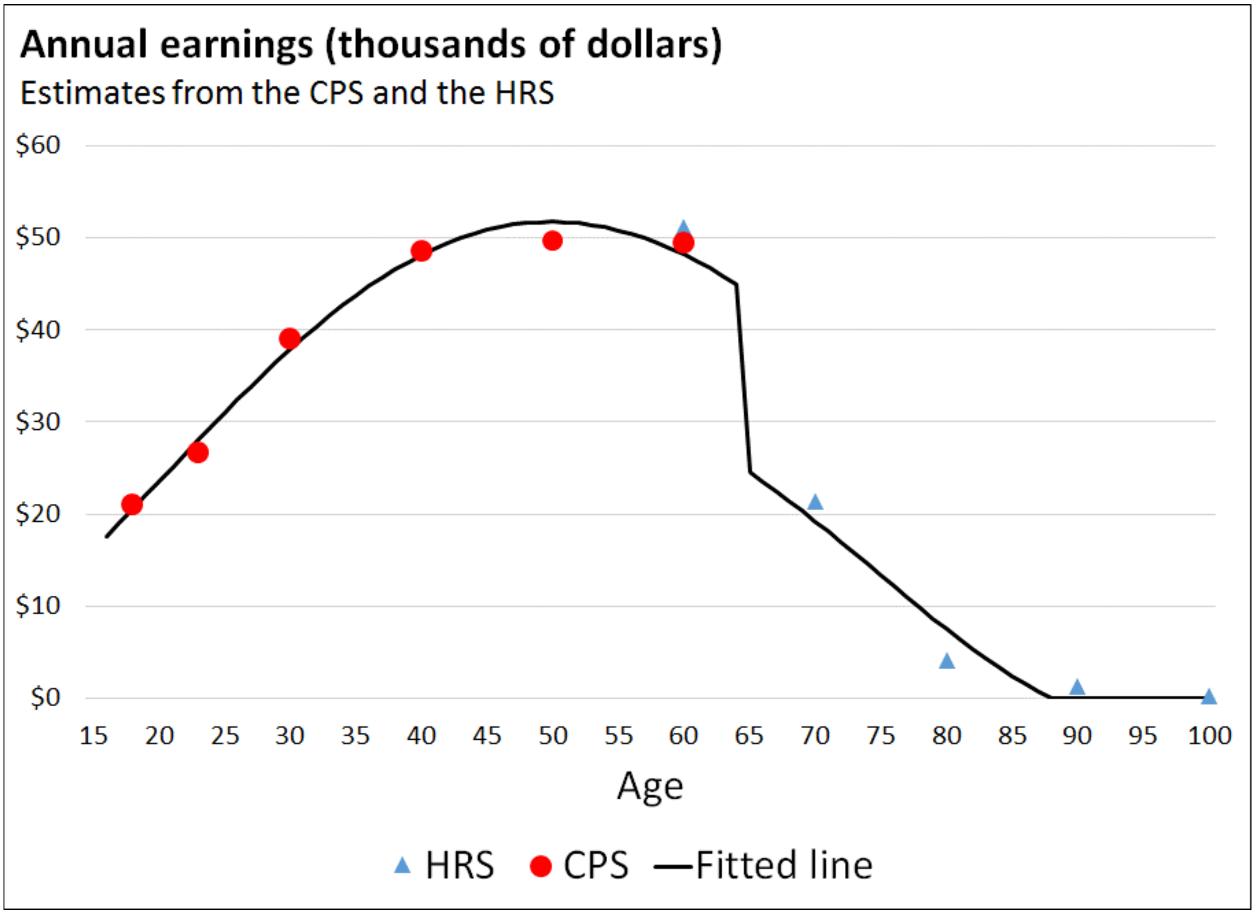
B1. Earnings

We obtain earnings data for employed individuals under the age of 65 from the 2016 Current Population Survey (CPS).[31] We also obtain earnings data for respondents over the age of 55 from the 2014 Health and Retirement Survey (HRS). For both surveys, the data represent earnings before taxes and other deductions, and include wages, salaries, and tips. The HRS earnings data also include self-employment income. (The CPS data exclude self-employed individuals.)

The CPS earnings data are binned into the following age groups: 16-19, 20-24, 25-34, 35-44, 45-54, and 55-64. We collapse the HRS earnings data into the following age groups: 55-64, 65-74, 75-84, 85-94, and 95-104. The resulting estimates are plotted in Appendix Figure B1. We smooth the data by fitting it to a quartic polynomial, and include an indicator variable for ages over 65. The dependent variable in the regression is the CPS earnings estimate for ages under 65, and the HRS estimate for ages over 65. Finally, we constrain the fitted prediction to be non-negative.

[31] These data are available at http://data.bls.gov/pdq/querytool.jsp?survey=le.

Appendix Figure B1. Annual earnings estimates from CPS and HRS



Notes: This figure plots annual earnings by midpoint of age group as estimated by the 2016 Current Population Survey (CPS) for respondents under age 65, and as estimated by the 2014 Health and Retirement Survey (HRS) for respondents over age 55. The fitted line corresponds to a regression of annual earnings on a quartic polynomial in age and an indicator equal to 1 for ages 65 and over. The dependent variable in that regression, annual earnings, corresponds to CPS estimates for ages under 65 and HRS estimates for ages over 65.

B2. Mortality, quality of life, and medical spending

We obtain data on mortality, quality of life, and medical spending by health state from the Future Elderly Model (FEM). The FEM follows Americans aged 50 years and older and projects their health and medical spending over time. A complete technical document detailing the FEM is available online.[32] The FEM is a microsimulation that follows the evolution of individual-level health trajectories and economic outcomes, rather than the average or aggregate characteristics of a cohort. The FEM has three core modules. The first is the Replenishing Cohorts module, which predicts economic and health outcomes of new cohorts of 50-year-olds with data from the Panel Study of Income Dynamics (PSID), and incorporates trends in disease and trends in other outcomes based on data from external sources, such as the National Health Interview Survey and the American Community Survey. This module generates cohorts as the simulation proceeds, so that we can measure outcomes for the age 50+ population in any given year.

[32] A complete technical description is available at roybalhealthpolicy.usc.edu/fem/technical-specifications/.

The second component is the Health Transition module, which uses the longitudinal structure of the Health and Retirement Survey (HRS) to calculate transition probabilities across various health states, including chronic conditions, functional status, body-mass index, and mortality, using linear and nonlinear multivariate regression models. These transition probabilities depend on a battery of predictors: age, sex, education, race, ethnicity, smoking behavior, marital status, employment and health conditions. Baseline factors are also controlled for using a series of initial health variables measured at age 50. FEM transitions produce a large set of simulated outcomes, including diabetes, high-blood pressure, heart disease, cancer (except skin cancer), stroke or transient ischemic attack, and lung disease (either or both chronic bronchitis and emphysema), disability, and body-mass index. Disability is measured by limitations in instrumental activities of daily living, activities of daily living, and residence in a nursing home. This dynamic simulation method has undergone extensive benchmarking and validation.

Finally, the Policy Outcomes module combines individual-level outcomes into aggregate outcomes, such as medical care costs (Medicare, Medicaid and Private), federal, state and property taxes, Social Security expenditures and contributions. Individual health spending is predicted with regard to health status (chronic conditions and functional status), demographics (age, sex, race, ethnicity and education), nursing home status, and mortality. Estimates are based on spending data from the Medical Expenditure Panel Survey for individuals aged 64 and younger and the Medicare Current Beneficiary Survey for individuals aged 65 and older, who constitute the bulk of the Medicare population. This module has been comprehensively tested against national aggregates.

An example of how the three modules interact is as follows. For year 2014, the model begins with the population of Americans aged 50 and older based on nationally representative data from the HRS. Individual-level health and economic outcomes for the next two years are predicted using the Policy Outcomes module. The cohort is then aged two years using the Health Transition Module. Aggregate health and functional status outcomes for those years are then calculated. At that point, a new cohort of 50-year-olds is introduced into the 2016 population using the Replenishing Cohort module, and they join those who survived from 2014 to 2016. This forms the age 50+ population for 2016. The transition model is then applied to this population. The same process is repeated until reaching the last year of the simulation.

C. Supporting calculations for numerical models

Appendix C1 provides details regarding the implementation of the deterministic mortality model employed in Section IV.C, and explains how it is used to derive the aggregate insurance value of Social Security. This model is estimated numerically using standard dynamic programming methods.

Appendix C2 provides a derivation of the stochastic mortality model employed in Section IV.B. This model is solved analytically and thus provides exact solutions.

C1. Deterministic mortality

In this model, there is only one health state and we abstract from quality of life. The optimal value function then simplifies to:

$$V\big(t, W(t)\big) = \max_{c(t)} \sum_{s=t}^{T} e^{-\rho(s-t)} S_t(s) u(c(s))$$

We can use the value function to rewrite the optimization problem as a recursive Bellman equation:

$$V\big(t, W(t)\big) = \max_{c(t)} u\big(c(t)\big) + \frac{1 - d(t)}{e^{\rho}} V\big(t+1, W(t+1)\big)$$

Because the problem is finite, we can work backwards from the final period. We discretize the state space into $N_w = 3{,}000$ points evenly distributed across the interval $[0, W_{max}]$. Let that set of values be $\{W_n\}$. Define $g_t\big(W(t)\big) = W(t+1)$ as a mapping from the current wealth state, $W(t)$, to the optimal wealth state in the following period, $W(t+1)$.

It is clear that the consumer should consume all her wealth in the final period, i.e., $g_T\big(W(T)\big) = 0$ for all $W(T) \in \{W_n\}$. This implies that $V\big(T, W(T)\big) = u\big(W(T) + y(T)\big)$ for all $W(T) \in \{W_n\}$.

Next, we calculate $V(T-1, W_{T-1}) = \max_{g(W_{T-1}) = W_T} u(W_{T-1} + y(T-1) - W_T/e^r) + \frac{1-d(T-1)}{e^{\rho}} V(T, W_T)$. In other words, for each $W(T-1) \in \{W_n\}$, we calculate the optimal $V\big(T-1, W(T-1)\big)$ by determining which choice of $g_{T-1}\big(W(T-1)\big) = W(T) \in \{W_n\}$ will maximize utility. This algorithm is then repeated for $t = T-2, T-3, \dots, 1$.

Given the initial condition, W_1, we can then employ our results to calculate $W(2) = g_1\big(W(1)\big)$, $W(3) = g_2\big(W(2)\big)$,…, $W(T) = g_{T-1}\big(W(T-1)\big)$. Period consumption, $c(t)$, is then calculated using the equation for the budget constraint. Finally, we use the analytical formulas derived in Section II to calculate the value of statistical life.

When accounting for a bequest motive, we follow Kopczuk and Lupton (2007) and assume the utility from leaving a bequest is linear in wealth:

$$V\big(t, W(t)\big) = \max_{c(t)} u\big(c(t)\big) + \frac{1}{e^{\rho}} \Big[\big(1 - d(t)\big) V\big(t+1, W(t+1)\big) + d(t) \alpha W(t+1)\Big]$$

Kopczuk and Lupton (2007) estimate that the constant $\alpha^{-\gamma}$ is approximately equal to \$50,000, where γ is the coefficient of relative risk aversion from a CRRA utility function. We adopt a (stronger) estimate of \$35,000 when accounting for a bequest motive. This parameterization implies that the marginal utility of consumption is less than the marginal utility of leaving a bequest when consumption in the last year of life is more than \$35,000.

Insurance value of Social Security

We calculate the insurance value of Social Security at all ages by estimating its wealth equivalence. That is, we follow Mitchell et al. (1999) and estimate the amount of wealth, W^*, required to equalize the utilities of a non-annuitized individual and an individual with Social Security. In other words, we solve for compensating wealth at age t, $W^*(t)$, such that $V\big(t, W(t) + W^*(t)\big) = V^{SS}\big(t, W^{SS}(t)\big)$. Wealth for a non-annuitized individual, $W(t)$, and wealth for an individual with Social Security, $W^{SS}(t)$, are calculated by the deterministic model for the first two policy scenarios discussed in the main text.

We solve for $W^*(t)$ by applying a numerical search algorithm. We estimate that, at age 65, having access to Social Security is equivalent to an increase in wealth of 16.5 percent for a non-annuitized individual. By way of comparison, Mitchell et al. (1999) estimate the before-tax value of *full* (complete) annuitization at age 65 to be 37.4 percent of wealth, using the same parameters for risk aversion, interest rate, and the discount rate.

The aggregate insurance value of Social Security is then calculated by aggregating over the 2015 US population:

$$Aggregate\ Value\ SS = \sum_{a=0}^{110} W^*(a) f(a)$$

C2. Stochastic mortality

We focus on the case where the consumer does not have access to annuities. We assume that the consumer's lifetime wealth is available at time $t = 0$, so that we can abstract away from the income-generating process. This allows us to generate an analytic solution to the consumer's problem, given by:

$$\max_{c(t)} \mathbb{E}\left[\sum_{t=0}^{T} e^{-\rho t} S_0(t) u\left(c(t), q_{Y_t}(t)\right) + e^{-\rho(t+1)}\left(\big(S_0(t) - S_0(t+1)\big) u(W(t+1), b_t)\right) \middle| Y_0, W_0\right]$$

subject to:

$$W(0) = W_0,$$
$$W(t) \geq 0,$$
$$W(t+1) = \big(W(t) - c(t)\big) e^{r(t, Y_t)}$$

Here, Y_t denotes the consumer's health state at time t, and we allow the interest rate to depend on it so as to model health-related wealth shocks, as described in the main text. Of course, a constant interest rate $r(t, i) = r$ is included as a special case. The parameter b_t measures the bequest motive. The utility function is:

$$u(c, q) = q \frac{c^{1-\gamma}}{1-\gamma} - \frac{\underline{c}^{1-\gamma}}{1-\gamma}$$

where $\underline{c}$ is the subsistence level of consumption for a healthy person. Because optimal consumption is unaffected by affine transformations of utility, we will assume $u(c, q) = qc^{1-\gamma}/(1-\gamma)$ when solving the model for consumption.

Define the value function as:

$$V(t, W(t), Y_t) = \max_{c(s)} \mathbb{E}\left[\sum_{s=t}^{T} e^{-\rho(s-t)} S_t(s) u\left(c(s), q_{Y_s}(s)\right) + e^{-\rho(s+1-t)}\big(S_t(s) - S_t(s+1)\big) u(W(s+1), b_s) \middle| Y_t, W(t)\right]$$

subject to:

$$W(s+1)=\big(W(s)-c(s)\big)e^{r(s,Y_s)}, s>t, W(s)\geq 0$$

Then we obtain the following Bellman equation:

$$V(t,w,i)=\max_{c(t)}\Bigg\{u\big(c(t),q_i(t)\big)+e^{-\rho}\overline{d}_i(t)u\left(\big(w-c(t)\big)e^{r(t,i)},b_t\right) + e^{-\rho}\left(1-\overline{d}_i(t)\right)\sum_{j=1}^{n}p_{ij}(t)V\big(t+1,\big(w-c(t)\big)e^{r(t,i)},j\big)\Bigg\}$$

Appendix Proposition C1:

The value function and the optimal consumption level satisfy:

$$V(t,w,i)=\frac{w^{1-\gamma}}{1-\gamma}K_{t,i},$$

$$c^*(t,w,i)=w\cdot c_{t,i}$$

where:

$$c_{t,i}=\left[1+e^{-r(t,i)}\left(\frac{e^{r(t,i)}\left[\overline{d}_i(t)b_t+\left(1-\overline{d}_i(t)\right)\left(\sum_{j=1}^{n}p_{ij}(t)K_{t+1,j}\right)\right]}{e^{\rho}q_i(t)}\right)^{\frac{1}{\gamma}}\right]^{-1}, t<T,$$

$$c_{T,i}=\left[1+e^{-r(T,i)}\left(\frac{e^{r(T,i)}b_T}{e^{\rho}q_i(T)}\right)^{\frac{1}{\gamma}}\right]^{-1}$$

and $K_{t,i}$ satisfies the recursion:

$$K_{t,i}=\left[q_i(t)^{\frac{1}{\gamma}}+e^{-r(t,i)}\left[e^{r(t,i)-\rho}\left(\overline{d}_i(t)b_t+\left(1-\overline{d}_i(t)\right)\left(\sum_{j=1}^{n}p_{ij}(t)K_{t+1,j}\right)\right)\right]^{\frac{1}{\gamma}}\right]^{\gamma}, t<T,$$

$$K_{T,i}=\left[q_i(T)^{\frac{1}{\gamma}}+e^{-r(T,i)}\left(e^{r(T,i)-\rho}b_T\right)^{\frac{1}{\gamma}}\right]^{\gamma}$$

Proof of Appendix Proposition C1: see end of appendix C

When calculating VSL, we incorporate subsistence consumption back into the utility function. In this case, the value function is:

$$V(0,w,i)=\sum_{t=0}^{T}e^{-\rho t}\mathbb{E}\left[\exp\left\{-\int_0^t\mu(s)ds\right\}\left(q_{Y_t}(t)\frac{c(t)^{1-\gamma}}{1-\gamma}-\frac{\underline{c}^{1-\gamma}}{1-\gamma}\right)\middle|Y_0=i,W(0)=w\right] + \underbrace{e^{-\rho(t+1)}\mathbb{E}\left[\left(\exp\left\{-\int_0^t\mu(s)ds\right\}-\exp\left\{-\int_0^{t+1}\mu(s)ds\right\}\right)\left(b_t\frac{W(t+1)^{1-\gamma}}{1-\gamma}-\frac{\underline{c}^{1-\gamma}}{1-\gamma}\right)\middle|Y_0=i,W(0)=w\right]}_{*} \quad \text{(C1)}$$

In specifications without the bequest motive, the second term (*) is dropped. Rearranging yields:

$$
\begin{aligned}
V(0,w,i) = \sum_{t=0}^{T} e^{-\rho t}\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\} q_{Y_t}(t)\frac{c(t)^{1-\gamma}}{1-\gamma}\middle| Y_0 = i, W(0) = w\right] \\
+ e^{-\rho(t+1)} b_t \mathbb{E}\left[\left(\exp\left\{-\int_0^t \mu(s)ds\right\} - \exp\left\{-\int_0^{t+1} \mu(s)ds\right\}\right)\frac{W(t+1)^{1-\gamma}}{1-\gamma}\middle| Y_0 = i, W(0) = w\right] \\
- \frac{\underline{c}^{1-\gamma}}{1-\gamma}\left[1 + e^{-\rho}\sum_{t=0}^{T} e^{-\rho t}\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\}\middle| Y_0 = i\right]\right] \\
= \frac{1}{1-\gamma}\left[w^{1-\gamma}K_{0,i} - \underline{c}^{1-\gamma}\left[1 + e^{-\rho}\underbrace{\sum_{t=0}^{T} e^{-\rho t}\,\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\}\middle| Y_0 = i\right]}_{\text{life expect. in state } i,\ \text{discounted at rate } \rho}\right]\right]
\end{aligned}
$$

We can then calculate VSL in state i using the following formula:

$$
VSL(i) = \frac{V(0,w,i)}{u_c\left(wc_{0,i}, q_i(0)\right)} = \frac{V(0,w,i)}{V_w(0,w,i)} \tag{C2}
$$

When bequests are absent and $r(t,i) = r$, we drop the term (*) in equation (C1), and the theory presented in the main text then yields the following expression for VSL:

$$
\begin{aligned}
VSL(i) &= \mathbb{E}\left[\sum_{t=0}^{T}\exp\left\{-\int_0^t \rho + \mu(s)ds\right\}\frac{u\left(c(t), q_{Y_t}(t)\right)}{u_c\left(c(0), q_{Y_0}(0)\right)}\middle| Y_0 = i, W(0) = w\right] \\
&= \sum_{t=0}^{T} e^{-rt}\frac{\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\} u\left(c(t), q_{Y_t}(t)\right)\middle| Y_0 = i, W(0) = w\right]}{\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\} u_c\left(c(t), q_{Y_t}(t)\right)\middle| Y_0 = i, W(0) = w\right]} \\
&= \sum_{t=0}^{T} e^{-rt}\frac{\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\}\left(q_{Y_t}(t)\frac{c(t)^{1-\gamma}}{1-\gamma} - \frac{\underline{c}^{1-\gamma}}{1-\gamma}\right)\middle| Y_0 = i, W(0) = w\right]}{\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\} q_{Y_t}(t)c(t)^{-\gamma}\middle| Y_0 = i, W(0) = w\right]}
\end{aligned}
$$

which can also be written as:

$$
VSL(i) = \frac{1}{1-\gamma}\sum_{t=0}^{T} e^{-rt}\underbrace{\frac{\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\} q_{Y_t}(t)c(t)^{1-\gamma}\middle| Y_0 = i, W(0) = w\right] - \underline{c}^{1-\gamma}\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\}\middle| Y_0 = i, W(0) = w\right]}{\mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\} q_{Y_t}(t)c(t)^{-\gamma}\middle| Y_0 = i, W(0) = w\right]}}_{v(Y_0,t)} \tag{C3}
$$

To evaluate this expression for VSL, we will make use of the following lemma.

Appendix Lemma C2: Let $W_{t,j}(\Psi) = \mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\} W(t)^{\Psi}\mathbf{1}\{Y_t = j\}\middle| Y_0, W_0\right]$ for $\Psi \in (1,\infty)$. Then $W_{t,j}(\Psi)$ satisfies the following recursion:

$$
\begin{aligned}
&W_{0,Y_0}(\Psi) = W_0^{\Psi}, W_{0,i}(\Psi) = 0, i \neq Y_0, \\
&W_{t+1,j}(\Psi) = e^{r\Psi}\sum_{k=1}^{n} W_{t,k}(\Psi)\left(1 - c_{t,k}\right)^{\Psi}\left(1 - \overline{d}_k(t)\right)p_{k,j}(t)
\end{aligned}
$$

Proof of Appendix Lemma C2: see end of appendix C

Note that for $\Psi = 0$, the expression $\sum_{j=1}^{n} W_{t,j}(0) = \mathbb{E}\left[\exp\left\{-\int_0^t \mu(s)ds\right\}\middle| Y_0\right]$ is simply the t-year survival probability. Applying **Appendix Lemma C2**, we obtain:

Appendix Proposition C3:

VSL in state Y_0 is equal to:

$$VSL(Y_0) = \frac{1}{1-\gamma}\sum_{t=0}^{T} e^{-rt} \underbrace{\frac{\sum_{j=1}^{n} q_j(t) c_{t,j}^{1-\gamma} W_{t,j}(1-\gamma) - \underline{c}^{1-\gamma} \sum_{j=1}^{n} W_{t,j}(0)}{\sum_{j=1}^{n} q_j(t) c_{t,j}^{-\gamma} W_{t,j}(-\gamma)}}_{v(Y_0,t)}$$

Proof of Appendix Proposition C3: see end of appendix C

We also immediately obtain the following corollary.

Appendix Corollary C4:

The value of a marginal reduction in the probability of transitioning from state i to state j is equal to:

$$\begin{aligned} VSI(i,j) &= VSL(i) - VSL(j)\frac{q_j(0)c_{0,j}^{-\gamma}}{q_i(0)c_{0,i}^{-\gamma}} \\ &= VSL(i) - \left(\frac{q_j(0)}{q_i(0)}\right)\left(\frac{c_{0,i}}{c_{0,j}}\right)^{\gamma} VSL(j) \end{aligned}$$

We have verified in our numerical calculations that **Appendix Proposition C3** and **Appendix Corollary C4** yield the same answer as the direct evaluation via equation (C2) above.

Proofs for Appendix C

Proof of Appendix Proposition C1:

The proof proceeds by induction on $t \le T$. For the base case $t = T$, note that $\overline{d}_i(t) = 1$, so that the first-order condition from the Bellman equation gives:

$$q_i(T)c(T)^{-\gamma} = e^{r(T,i)-\rho} b_T \big(w - c(T)\big)^{-\gamma} e^{-r(T,i)\gamma}$$

This implies that:

$$\begin{aligned} c(T) &= \frac{w e^{r(T,i)} e^{\frac{(\rho - r(T,i))}{\gamma}} \left(\frac{q_i(T)}{b_T}\right)^{\frac{1}{\gamma}}}{1 + e^{r(T,i)} e^{\frac{(\rho - r(T,i))}{\gamma}} \left(\frac{q_i(T)}{b_T}\right)^{\frac{1}{\gamma}}} \\ &= w \underbrace{\left[1 + e^{-r(T,i)} \left(\frac{e^{r(T,i)} b_T}{e^{\rho} q_i(T)}\right)^{\frac{1}{\gamma}}\right]^{-1}}_{c_{T,i}} \end{aligned}$$

Hence, we obtain:

$$\begin{aligned} V(T,w,i) &= \frac{w^{1-\gamma}}{1-\gamma}\Big(q_i(T) c_{T,i}^{1-\gamma} + e^{-\rho} b_T e^{r(T,i)(1-\gamma)} \big(1 - c_{T,i}\big)^{1-\gamma}\Big) \\ &= \frac{e^{-\rho} e^{r(T,i)(1-\gamma)}}{\left[b_T^{\frac{1}{\gamma}} + e^{r(T,i)} e^{\frac{(\rho - r(T,i))}{\gamma}} q_i(T)^{\frac{1}{\gamma}}\right]^{-\gamma}} \\ &= \left[q_i(T)^{\frac{1}{\gamma}} + e^{-r(T,i)} \big(e^{(r(T,i)-\rho)} b_T\big)^{\frac{1}{\gamma}}\right]^{\gamma} \end{aligned}$$

For the induction step, suppose the proposition is true for case $t + 1$. We have:

$$V(t,w,i) = \max_c \left\{ q_i(t) \frac{c^{1-\gamma}}{1-\gamma} + b_t e^{-\rho} \overline{d}_i(t) \frac{\big((w-c)e^{r(t,i)}\big)^{1-\gamma}}{1-\gamma} + e^{-\rho} \Big(1 - \overline{d}_i(t)\Big) \sum_{j=1}^{n} p_{ij}(t) \frac{K_{t+1,j}}{1-\gamma} \big[(w-c)e^{r(t,i)}\big]^{1-\gamma} \right\}$$

From the first-order condition we obtain:

$$q_i(t) c^{-\gamma} = b_t e^{r(t,i)-\rho} \overline{d}_i(t) e^{-r(t,i)\gamma} (w-c)^{-\gamma} + e^{r(t,i)-\rho} \Big(1 - \overline{d}_i(t)\Big) e^{-\gamma r(t,i)} (w-c)^{-\gamma} \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j}$$

Rearranging yields:

$$q_i(t) c^{-\gamma} = (w-c)^{-\gamma} e^{r(t,i)-\rho} e^{-r(t,i)\gamma} \left[\overline{d}_i(t) b_t + \Big(1 - \overline{d}_i(t)\Big) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j}\right]$$

which implies:

$$q_i(t)^{-1/\gamma} c = (w - c) e^{(\rho - r(t,i))/\gamma} e^{r(t,i)} \left[\overline{d}_i(t) b_t + \left(1 - \overline{d}_i(t)\right) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j} \right]^{-1/\gamma}$$

Rearranging further yields:

$$\begin{aligned} c &= w \frac{e^{r(t,i)} \left[e^{r(t,i)} \left[\overline{d}_i(t) b_t + \left(1 - \overline{d}_i(t)\right) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j} \right] \right]^{-1/\gamma}}{e^{\rho} q_i(t)^{-1/\gamma} + e^{r(t,i)} \left[e^{r(t,i)} \left[\overline{d}_i(t) b_t + \left(1 - \overline{d}_i(t)\right) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j} \right] \right]^{-1/\gamma}} \\ &= w \underbrace{\left[1 + e^{-r(t,i)} \left(\frac{e^{r(t,i)} \left[\overline{d}_i(t) b_t + \left(1 - \overline{d}_i(t)\right) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j} \right]}{e^{\rho} q_i(t)} \right)^{\frac{1}{\gamma}} \right]^{-1}}_{c_{t,i}} \end{aligned}$$

Thus we obtain:

$$V(t,w,i) = q_i(t) c_{t,i}^{1-\gamma} \frac{w^{1-\gamma}}{1-\gamma} + b_t e^{-\rho} \overline{d}_i(t) \frac{w^{1-\gamma}}{1-\gamma} \left(1 - c_{t,i}\right)^{1-\gamma} e^{r(t,i)(1-\gamma)} + e^{-\rho} \left(1 - \overline{d}_i(t)\right) \frac{w^{1-\gamma}}{1-\gamma} \left(1 - c_{t,i}\right)^{1-\gamma} e^{r(t,i)(1-\gamma)} \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j}$$

$$= \frac{w^{1-\gamma}}{1-\gamma} \left[q_i(t) c_{t,i}^{1-\gamma} + e^{-\rho} \left(1 - c_{t,i}\right)^{1-\gamma} e^{r(t,i)(1-\gamma)} \left[\overline{d}_i(t) b_t + \left(1 - \overline{d}_i(t)\right) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j} \right] \right]$$

$$= \frac{w^{1-\gamma}}{1-\gamma} \frac{q_i(t) e^{r(t,i)(1-\gamma)} \left[e^{r(t,i)} \left(\overline{d}_i(t) b_t + \left(1 - \overline{d}_i(t)\right) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j} \right) \right]^{1-1/\gamma} + e^{-\rho} e^{r(t,i)(1-\gamma)} \left(e^{\rho} q_i(t)\right)^{1-1/\gamma} \left[\overline{d}_i(t) b_t + \left(1 - \overline{d}_i(t)\right) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j} \right]}{\left[\left(e^{\rho} q_i(t)\right)^{-1/\gamma} + e^{r(t,i)} \left[e^{r(t,i)} \left[\overline{d}_i(t) b_t + \left(1 - \overline{d}_i(t)\right) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j} \right] \right]^{-\frac{1}{\gamma}} \right]^{1-\gamma}}$$

$$= \frac{w^{1-\gamma}}{1-\gamma} \frac{e^{r(t,i)(1-\gamma)} q_i(t) \left[\overline{d}_i(t) b_t + \left(1 - \overline{d}_i(t)\right) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j} \right]}{\left[\left(e^{\rho} q_i(t)\right)^{-1/\gamma} + e^{r(t,i)} \left[e^{r(t,i)} \left[\overline{d}_i(t) b_t + \left(1 - \overline{d}_i(t)\right) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j} \right] \right]^{-\frac{1}{\gamma}} \right]^{-\gamma}}$$

$$= \frac{w^{1-\gamma}}{1-\gamma} \underbrace{\left[q_i(t)^{\frac{1}{\gamma}} + e^{-r(t,i)} \left[e^{r(t,i)-\rho} \left(\overline{d}_i(t) b_t + \left(1 - \overline{d}_i(t)\right) \sum_{j=i}^{n} p_{ij}(t) K_{t+1,j} \right) \right]^{\frac{1}{\gamma}} \right]^{\gamma}}_{K_{t,i}}$$

QED

Proof of Appendix Lemma C2:

$$\begin{aligned} W_{t+1,j}(\Psi) &= \mathbb{E} \left[\exp\left\{ -\int_0^{t+1} \mu(s) ds \right\} \left(W(t+1)\right)^{\Psi} \mathbf{1}\{Y_{t+1} = j\} \middle| Y_0, W_0 \right] \\ &= \mathbb{E} \left[\exp\left\{ -\int_0^{t} \mu(s) ds \right\} \left(\left(W(t) - c(t)\right) e^{r} \right)^{\Psi} \mathbf{1}\{Y_{t+1} = j\} \exp\left\{ -\int_t^{t+1} \mu(s) ds \right\} \middle| Y_0, W_0 \right] \end{aligned}$$

$$= \sum_{k=1}^{n} \mathbb{E}\left[\mathbf{1}\{Y_t = k\} \exp\left\{ -\int_0^t \mu(s) ds \right\} e^{r\Psi} W(t)^{\Psi} \left(1 - c_{t,k}\right)^{\Psi} \underbrace{\mathbb{E}\left[\mathbf{1}\{Y_{t+1} = j\} \exp\left\{ -\int_t^{t+1} \mu(s) ds \right\} \middle| Y_t = k \right]}_{\left(1-\overline{d}_k(t)\right) p_{kj}(t)} \middle| Y_0, W_0 \right]$$

$$= e^{r\Psi} \sum_{k=1}^{n} W_{t,k}(\Upsilon) \left(1 - c_{t,k}\right)^{\Psi} \left(1 - \overline{d}_k(t)\right) p_{kj}(t)$$

QED

Proof of Appendix Proposition C3:

Note that we can rewrite one of the terms in equation (C3) as follows:

$$\mathbb{E}\left[\exp\left\{ -\int_0^t \mu(s) ds \right\} q_{Y_t}(t) c(t)^{\Psi} \middle| Y_0, W_0 \right] = \sum_{j=1}^{n} \mathbb{E}\left[\exp\left\{ -\int_0^t \mu(s) ds \right\} q_{Y_t}(t) c(t)^{\Psi} \mathbf{1}\{Y_t = j\} \middle| Y_0, W_0 \right]$$

$$= \sum_{j=1}^{n} \mathbb{E}\left[\exp\left\{ -\int_0^t \mu(s) ds \right\} q_j(t) c_{t,j}^{\Psi} W(t)^{\Psi} \mathbf{1}\{Y_t = j\} \middle| Y_0, W_0 \right]$$

$$= \sum_{j=1}^{n} q_j(t) c_{t,j}^{\Psi} \underbrace{\mathbb{E}\left[\exp\left\{ -\int_0^t \mu(s) ds \right\} W(t)^{\Psi} \mathbf{1}\{Y_t = j\} \middle| Y_0, W_0 \right]}_{W_{t,j}(\Psi)}$$

The proof follows by setting $\Psi = 1 - \gamma$, 0, and $-\gamma$ and then plugging those results into equation (C3) as appropriate.

QED

D. The fully annuitized value of life when mortality is stochastic

We assume a full menu of actuarially fair annuities is available, where consumers can choose consumption streams, $c(t)$, that depend on the evolution of their health state. Thus, the consumer is able to fully insure against consumption risk. The consumer's maximization problem is:

$$\max_{c(t)} \mathbb{E}\left[\int_0^T e^{-\rho t} S(t) u\big(c(t), q_{Y_t}(t)\big) dt \,\middle|\, Y_0\right] \tag{D1}$$

subject to:

$$\mathbb{E}\left[\int_0^T e^{-rt} S(t) c(t) dt \,\middle|\, Y_0\right] = W_0 + \mathbb{E}\left[\int_0^T e^{-rt} S(t) m_{Y_t}(t) dt \,\middle|\, Y_0\right] \equiv \overline{W}(0, Y_0)$$

where $\overline{W}(0, Y_0)$ is the net present value of wealth and future earnings.

The consumer chooses the consumption profile at time t based on her health state, $Y_t = i$, and on her available wealth, $\overline{W}(t, i)$. Her available wealth finances future consumption such that:

$$\overline{W}(t, i) = \mathbb{E}\left[\int_t^T e^{-r(u-t)} \exp\left\{-\int_t^u \mu(s) ds\right\} c(u) du \,\middle|\, Y_t, \overline{W}(t, i)\right]$$

Appendix Lemma D1:

The law of motion for wealth is:

$$\frac{\partial \overline{W}(t, i)}{\partial t} = \big(r + \bar{\mu}_i(t)\big) \overline{W}(t, i) - c\big(t, \overline{W}(t, i), i\big) + \sum_{j>i} \lambda_{ij}(t) \big[\overline{W}(t, i) - \overline{W}(t, j)\big], i = 1, \dots, n$$

Proof of Appendix Lemma D1: see end of Appendix D

Note that the dynamics for $\overline{W}(t, i)$ will depend on $\overline{W}(t, j)$, $j > i$, so that $\big(Y_t, \overline{W}(t, Y_t)\big)$ is not Markov, but $\big(Y_t, \overline{W}(t)\big)$, where we define the wealth vector $\overline{W}(t) \equiv \big(\overline{W}(t, 1), \dots, \overline{W}(t, n)\big)$, is Markov.

Define the optimal value-to-go function as:

$$V\big(t, \overline{W}(t), Y_t\big) = \max_{c(u)} \mathbb{E}\left[\int_t^T e^{-\rho(u-t)} \exp\left\{-\int_t^u \mu(s) ds\right\} u\big(c(u), q_{Y_u}(u)\big) du \,\middle|\, Y_t, \overline{W}(t)\right]$$

subject to the law of motion for wealth given above. As a stochastic dynamic programming problem, $V(\cdot)$ satisfies the following Hamilton-Jacobi-Bellman (HJB) system of equations:

$$\begin{aligned}\big(\rho + \bar{\mu}_i(t)\big) V\big(t, \overline{W}(t), i\big) = \frac{\partial V\big(t, \overline{W}(t), i\big)}{\partial t} + \max_{c(t)} \Bigg\{ & u\big(c(t), q_i(t)\big) + \sum_{j>i} \lambda_{ij}(t) \big[V\big(t, \overline{W}(t), j\big) - V\big(t, \overline{W}(t), i\big)\big] \\ & + \sum_{k \geq i} \frac{\partial V\big(t, \overline{W}(t), i\big)}{\partial \overline{W}(t, k)} \Bigg[\big(r + \bar{\mu}_k(t)\big) \overline{W}(t, k) - c\big(t, \overline{W}(t, k), k\big) \\ & + \sum_{l>k} \lambda_{kl}(t) \big[\overline{W}(t, k) - \overline{W}(t, l)\big]\Bigg]\Bigg\}, 1 \leq i \leq n\end{aligned} \tag{D2}$$

Similarly to the uninsured case presented in the main text, we follow Parpas and Webster (2013) and focus on the path of Y that begins in i and remains in i until time t, with $c_i(t)$ and $\overline{W}_i(t)$ denoting the corresponding optimal consumption and wealth paths. We take optimal consumption rules and value functions from other states as exogenous. As in the uninsured case, this approach will allow us to apply the standard Pontryagin maximum principle and derive analytic expressions.

Appendix Lemma D2:

The optimal value function for $Y_0 = i$, $V(0, \overline{W}(0,i), i)$, for the following deterministic optimization problem also satisfies the HJB given by (D2), for each $i \in \{1, \ldots, n\}$:

$$V(0, \overline{W}(0,i), i) = \max_{c_i(t)} \left[\int_0^T e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V\left(t, \overline{W}_i(t), j\right) \right) dt \right] \quad \text{(D3)}$$

subject to:

$$\frac{\partial \overline{W}_i(t,j)}{\partial t} = \left(r + \overline{\mu}_j(t)\right) \overline{W}_i(t,j) - c\left(t, \overline{W}_i(t), j\right) + \sum_{k>j} \lambda_{jk}(t) \left[\overline{W}_i(t,j) - \overline{W}_i(t,k)\right], j \neq i$$

$$\frac{\partial \overline{W}_i(t,i)}{\partial t} = \left(r + \overline{\mu}_j(t)\right) \overline{W}_i(t,i) - c_i(t) + \sum_{k>i} \lambda_{ik}(t) \left[\overline{W}_i(t,i) - \overline{W}_i(t,k)\right]$$

where $V(t, \overline{W}_i(t), j)$ and $c(t, \overline{W}_i(t), j), j > i$, are taken as exogenous.

Proof of Appendix Lemma D2: see end of Appendix D

Following Bertsekas (2005), the Hamiltonian for the (deterministic) maximization problem (D3) is:

$$\begin{aligned} H\left(\overline{W}_i(t), c_i(t), p_i(t)\right) &= e^{-\rho t} \tilde{S}(i,t) \left(u(c_i(t), q_i(t)) + \sum_{j>i} \lambda_{ij}(t) V\left(t, \overline{W}_i(t), j\right) \right) \\ &+ \sum_{k>i} p_i(t,k) \left[\left(r + \overline{\mu}_k(t)\right) \overline{W}_i(t,k) - c\left(t, \overline{W}_i(t), k\right) + \sum_{l>k} \lambda_{kl}(t) \left[\overline{W}_i(t,k) - \overline{W}_i(t,l)\right] \right] \\ &+ p_i(t,i) \left[\left(r + \overline{\mu}_i(t)\right) \overline{W}_i(t,k) - c_i(t) + \sum_{l>i} \lambda_{il}(t) \left[\overline{W}_i(t,i) - \overline{W}_i(t,l)\right] \right] \end{aligned} \quad \text{(D4)}$$

where $p_i(t) = \left(p_i(t,1), \ldots, p_i(t,n)\right)$ is the vector of costate variables corresponding to wealth $\overline{W}_i(t)$.

Appendix Lemma D3:

We have that $p_i(t,i) = \theta e^{-\rho t} \tilde{S}(i,t)$ for θ independent of i, and $p_i(t,k) = 0, k \neq i$. The necessary first-order condition for consumption is:

$$e^{(r-\rho)t} u_c(c_i(t), q_i(t)) = \theta \quad \text{(D5)}$$

where $\theta = p_i(0,i) = \partial V\left(0, \overline{W}_i(0), i\right) / \partial \overline{W}(0,i)$ is the marginal utility of wealth.

Proof of Appendix Lemma D3: see end of Appendix D

To analyze the values of life and illness, let $\delta_{ij}(t), i, j \leq N$, be a perturbation on the transition intensity $\lambda_{ij}(t)$, and let $\delta_{i,N+1}(t)$ be a perturbation on the mortality rate, $\overline{\mu}_i(t)$, where $\sum_{j=i+1}^{N+1} \int_0^T \delta_{ij}(t) dt = 1$, and consider:

$$\tilde{S}^{\varepsilon}(i,t) = \exp\left[-\int_0^t \left(\overline{\mu}_i(s) - \varepsilon\delta_{i,N+1}(s)\right) + \sum_{j=i+1}^{N}\left(\lambda_{ij}(s) - \varepsilon\delta_{ij}(s)\right) ds\right], \text{where } \varepsilon > 0$$

Appendix Proposition D4:

The marginal utility of preventing an illness or death is given by:

$$\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T \left(\tilde{S}(i,t)\left\{e^{-\rho t}\left[u\left(c_i(t),q_i(t)\right) + \sum_{j>i}\lambda_{ij}(t)V\left(t,\overline{W}_i(t),j\right)\right] + \theta e^{-rt}\left[m_i(t) - c_i(t) - \sum_{j>i}\lambda_{ij}(t)\overline{W}_i(t,j)\right]\right\} - \tilde{S}(i,t)\sum_{j=i+1}^{N}\delta_{ij}(t)\left\{e^{-\rho t}V\left(t,\overline{W}_i(t),j\right) - \theta e^{-rt}\overline{W}_i(t,j)\right\}\right) dt \quad \text{(D6)}$$

Proof of Appendix Proposition D4: see end of Appendix D

To obtain the value of statistical life (VSL), we first set $\delta_{i,N+1}$ equal to the Dirac delta function, and set all other perturbations equal to 0. Dividing the result by the marginal utility of wealth, θ, then yields:

$$\begin{aligned} VSL &= \int_0^T \tilde{S}(i,t)e^{-rt}\left\{\left[\frac{u(c_i(t),q_i(t))}{u_c(c_i(t),q_i(t))} + \sum_{j>i}\lambda_{ij}(t)\frac{V\left(t,\overline{W}_i(t),j\right)}{\partial V\left(t,\overline{W}_i(t),j\right)/\partial \overline{W}_i(t,j)}\right] + \left[m_i(t) - c_i(t) - \sum_{j>i}\lambda_{ij}(t)\overline{W}_i(t,j)\right]\right\} dt \\ &= \mathbb{E}\left[\int_0^T e^{-rt}S(t)v(t)dt\right] \end{aligned} \quad \text{(D7)}$$

where the value of a statistical life-year is:

$$v(t) = \frac{u\left(c(t),q_{Y_t}(t)\right)}{u_c\left(c(t),q_{Y_t}(t)\right)} + m_{Y_t}(t) - c_{Y_t}(t)$$

Comparing (D7) to (3) reveals that generalizing the standard model to account for stochastic mortality alone does not alter the basic expression for VSL. Consumers continue to discount future life-years by the rate of interest and by survival. We can obtain the life-cycle profile of consumption in state i by differentiating the first-order condition (D5) with respect to t. Doing so confirms that, as in the deterministic case, annuitization insulates consumption from mortality risk:

$$\frac{\dot{c}_i(t)}{c_i(t)} = \sigma(r-\rho) + \sigma\eta\frac{\dot{q}}{q}$$

Our results demonstrate that stochastic mortality, by itself, does not alter the basic insights regarding VSL offered by the prior literature as long as one maintains the assumption of full annuitization.

However, a novel feature of the stochastic model is that it permits an investigation into the value of prevention. Inspecting the expression for the marginal utility of life extension (D6), the first term inside the integral represents the gain in marginal utility from a reduction in the probability of exiting state i. The second term represents the loss in marginal utility from the reduction in probability of transitioning to other possible states. The net effect depends on the consumer's marginal utility in the different states.

To analyze the value of prevention, consider a reduction in the transition probability for only one alternative state, j, so that $\delta_{ik}(t) = 0 \ \forall k \neq j$. The value of avoiding illness j is then equal to:

$$
\begin{aligned}
VSI(i,j) = & \int_0^T \tilde{S}(i,t)e^{-rt}\left\{\left[\frac{u(c_i(t),q_i(t))}{u_c(c_i(t),q_i(t))} + \sum_{j>i}\lambda_{ij}(t)\frac{V\big(t,\overline{W}_i(t),j\big)}{\partial V\big(t,\overline{W}_i(t),j\big)/\partial \overline{W}_i(t,j)}\right]\right. \\
& \left. + \left[m_i(t) - c_i(t) - \sum_{j>i}\lambda_{ij}(t)\overline{W}_i(t,j)\right]\right\}dt - \left[\frac{V\big(t,\overline{W}_i(t),j\big)}{\theta} - \overline{W}_i(0,j)\right] \\
= & \; VSL(i) - VSL\big(j\big|\overline{W}(0,j) = \overline{W}_i(0,j)\big)
\end{aligned} \tag{D8}
$$

Thus, equation (D8) demonstrates that $VSI(i,j)$ is equal to the difference in VSL for states i and j, with the caveat that VSL in state j uses a measure of total wealth evaluated from the perspective of a person in state i. This technicality arises because the value of the consumer's annuity depends on her expected survival. For example, an annuity is worth more to a healthy 65-year-old than it is to a 65-year-old who was just diagnosed with lung cancer.

Proofs for Appendix D

Proof of Appendix Lemma D1:

Available wealth can be written as:

$$\overline{W}(t,i)=\int_t^T \exp\left\{-\int_t^u r+\overline{\mu}_i(s)+\sum_{j>i}\lambda_{ij}(s)\,ds\right\}\left[c_i(t,u)+\sum_{j>i}\lambda_{ij}(u)\overline{W}_i(u,t,j)\right]du$$

where with a slight abuse of notation, $c_i(t,u)$ and $\overline{W}_i(u,t,j)$ denote the consumption and wealth paths for an individual who is in state i at time t and remains in state i until time u. The result then follows by taking the derivative with respect to t.

Proof of Appendix Lemma D2:

This proof follows the same logic as the proof of **Lemma 1** in Appendix A. Consider the deterministic optimization problem (D3). Denote the optimal value-to-go function as:

$$\overline{V}\big(t,\overline{W}_i(t),i\big)=\max_{c_i(t)}\left\{\int_t^T e^{-\rho u}\tilde{S}(i,u)\left(u(c_i(u),q_i(u))+\sum_{j>i}\lambda_{ij}(u)V\big(u,\overline{W}_i(u),j\big)\right)du\right\}$$

Setting $\overline{V}\big(t,\overline{W}_i(t),i\big)=e^{-\rho t}\tilde{S}(i,t)V\big(t,\overline{W}_i(t),i\big)$ then demonstrates that $V(\cdot)$ satisfies the HJB (D2) for i.

QED

Proof of Appendix Lemma D3:

The costate equations for the Hamiltonian (D4) are:

$$\dot{p}_i(t,i)=-\left[\big(r+\overline{\mu}_i(t)\big)+\sum_{l>i}\lambda_{il}(t)\right]p_i(t,i),$$

$$\dot{p}_i(t,k)=-\sum_{j>k}\lambda_{ij}(t)\frac{\partial V\big(t,\overline{W}_i(t),j\big)}{\partial\overline{W}_i(t,k)}+\sum_{k\geq j>i}p_i(t,j)\left(\frac{\partial c\big(t,\overline{W}_i(t),j\big)}{\partial\overline{W}_i(t,k)}+\lambda_{jk}(t)\right)$$
$$-\,p_i(t,k)\left[\big(r+\overline{\mu}_k(t)\big)+\sum_{l>k}\lambda_{kl}(t)\right]+p_i(t,i)\lambda_{ik}(t),\text{ for }k>i$$

From the first costate equation, we obtain:

$$p_i(t,i)=e^{-rt}\tilde{S}(i,t)\theta$$

Taking first-order conditions in the Hamiltonian (D4) and plugging this in then yields:

$$u_c\big(c_i(t),q_i(t)\big)=\frac{\partial V\big(t,\overline{W}_i(t),i\big)}{\partial\overline{W}_i(t,i)}=e^{(\rho-r)t}\theta$$

To see that this solution works, let θ be constant across states, and set $p_i(t,k)=0=\frac{\partial V(t,\overline{W}_i(t),i)}{\partial\overline{W}_i(t,k)}$. This then satisfies the costate equation system across i,k, and t. In particular, for the second equation we obtain

$$
\begin{aligned}
\dot{p}_i(t,k) &= -e^{-\rho t}\tilde{S}(i,t)\lambda_{ik}(t)\underbrace{\frac{\partial V\left(t,\overline{W}_i(t),k\right)}{\partial \overline{W}_i(t,k)}}_{e^{(\rho-r)t}\theta} + \lambda_{ik}(t)p_i(t,i) \\
&= 0
\end{aligned}
$$

QED

Proof of Appendix Proposition D4:

Starting from equation (D3), we have:

$$
\begin{aligned}
V^{\varepsilon}\left(0,\overline{W}_i(0,i),i\right) = \int_0^T e^{-\rho t}\exp\left\{-\int_0^t \overline{\mu}_i(s) + \sum_{j>i}\lambda_{ij}(s) - \varepsilon\sum_{j=i+1}^{N+1}\delta_{ij}(s)\,ds\right\}\Bigg[u\left(c_i^{\varepsilon}(t),q_i(t)\right) \\
+ \sum_{j=i+1}^{N}\left[\lambda_{ij}(t) - \varepsilon\delta_{ij}(t)\right]V\left(t,\overline{W}_i^{\varepsilon}(t),j\right)\Bigg]dt,
\end{aligned}
$$

where $c_i^{\varepsilon}(t)$ and $\overline{W}_i^{\varepsilon}(t)$ represent the equilibrium variations in $c_i(t)$ and $\overline{W}_i(t)$ caused by the perturbation, $\delta_{ij}(t)$. Differentiating then yields:

$$
\begin{aligned}
\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T e^{-\rho t}\tilde{S}(i,t)\left[u\left(c_i(t),q_i(t)\right) + \sum_{j=i+1}^{N}\lambda_{ij}(t)V\left(t,\overline{W}_i(t),j\right)\right]\left[\sum_{j=i+1}^{N+1}\int_0^t\delta_{ij}(s)ds\right] - e^{-\rho t}\tilde{S}(i,t)\sum_{j=i+1}^{N}\delta_{ij}(t)V\left(t,\overline{W}_i(t),j\right) \\
+ e^{-\rho t}\tilde{S}(i,t)\left[\underbrace{u_c\left(c_i(t),q_i(t)\right)}_{e^{-(r-\rho)t}\theta}\left.\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\right|_{\varepsilon=0} + \sum_{j=i+1}^{N}\underbrace{\frac{\partial V\left(t,\overline{W}_i(t),j\right)}{\partial \overline{W}_i(t,j)}}_{e^{-(r-\rho)t}\theta}\left.\frac{\partial \overline{W}_i(t,j)}{\partial \varepsilon}\right|_{\varepsilon=0}\right]dt
\end{aligned}
$$

Next, note that the budget constraint implies:

$$
\begin{aligned}
0 &= \left.\frac{\partial W_0}{\partial \varepsilon}\right|_{\varepsilon=0} \\
&= \frac{\partial}{\partial \varepsilon}\int_0^T e^{-rt}\exp\left\{-\int_0^t \overline{\mu}_i(s) + \sum_{j>i}\lambda_{ij}(s) - \varepsilon\sum_{j=i+1}^{N+1}\delta_{ij}(s)\,ds\right\}\Bigg(c_i^{\varepsilon}(t) - m_i(t) \\
&\qquad + \sum_{j=i+1}^{N}\left[\lambda_{ij}(t) - \varepsilon\delta_{ij}(t)\right]\overline{W}_i^{\varepsilon}(t,j)\Bigg)dt\Bigg|_{\varepsilon=0} \\
&= \int_0^T\Bigg(e^{-rt}\tilde{S}(i,t)\left[c_i(t) - m_i(t) + \sum_{j=i+1}^{N}\lambda_{ij}(t)\overline{W}_i(t,j)\right] - e^{-rt}\tilde{S}(i,t)\sum_{j=i+1}^{N}\delta_{ij}(t)\overline{W}_i(t,j) \\
&\qquad + e^{-rt}\tilde{S}(i,t)\left[\left.\frac{\partial c_i^{\varepsilon}(t)}{\partial \varepsilon}\right|_{\varepsilon=0} + \sum_{j=i+1}^{N}\lambda_{ij}(t)\left.\frac{\partial \overline{W}_i^{\varepsilon}(t,j)}{\partial \varepsilon}\right|_{\varepsilon=0}\right]\Bigg)dt
\end{aligned}
$$

Plugging this last result into the expression for $\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0}$ then yields the desired result for marginal utility:

$$\left.\frac{\partial V}{\partial \varepsilon}\right|_{\varepsilon=0} = \int_0^T \left(\tilde{S}(i,t) \left\{ e^{-\rho t} \left[u\big(c_i(t), q_i(t)\big) + \sum_{j=i+1}^{N} \lambda_{ij}(t) V\big(t, \overline{W}_i(t,j), j\big) \right] + \theta e^{-rt} \left[m_i(t) - c_i(t) - \sum_{j>i} \lambda_{ij}(t) \overline{W}_i(t,j) \right] \right\} - \tilde{S}(i,t) \left\{ e^{-\rho t} \sum_{j=1}^{N} \delta_{ij}(t) V\big(t, \overline{W}_i(t), j\big) - \theta e^{-rt} \sum_{j=i+1}^{N} \delta_{ij}(t) \overline{W}_i(t,j) \right\} \right) dt$$

QED